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Synergy of Modern Portfolio Theory and Cointegration

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Abstract
The most common method for building portfolios is modern portfolio theory. This theory has been criticised for relying on the correlation measure, which is a short-term measure. In order to address this critique, this study examines whether the Johansen cointegration test improves the efficiency of portfolios based on modern portfolio theory. For this purpose, the co-movements between ten cryptocurrencies and the Nifty 50 are examined, and the long-term portfolios are then optimized by removing the cointegrated assets from the portfolio for the period between April 2019 and March 2022. Due to the prevalence of naive investors in India, naive portfolios were also built and compared to optimal portfolios. The study concluded that the Johansen cointegration test enhances the efficiency of modern portfolio theory and naive portfolios do not overlap with modern portfolios.

Keywords: Portfolio optimization, Modern Portfolio Theory, Johansen Cointegration test, Cryptocurrencies.

JEL Classification: G11, C58

Introduction

A cryptocurrency is a system of instituting virtual "coins" and providing for their secure ownership and transaction using a cryptographic problem. (Harwick, 2016). Bitcoin (BTC), the first cryptocurrency, was first proposed in 2008 by a person or group of persons going by the nickname Satoshi Nakamoto (Nakamoto, n.d.). The emergence of this new concept is related to the epicentre of the global financial crisis of 2008–2010. The central banks started to significantly expand the monetary base (also known as "printing money") as an attempt to mitigate its effects, which decreased trust in the traditional fiat currencies. The foundation of the bitcoin protocol was a peer-to-peer network and the previously known public and private key cryptographic techniques, together with a new consensus mechanism called ‘‘Proof of Work’’. The concept behind bitcoin was to create a mechanism that would, for the first time in human history, allow anyone to trust another person and conduct online transactions without the need for a centralised management organisation (Wątorek et al., 2021). After the invention of bitcoin, there are a bunch of cryptocurrencies flowing in the digital market now (Rathore et al., 2022) that have attracted much attention in recent years (Akyildirim et al., 2021). As a result, tremendous numbers of investors surged into the cryptocurrency market in order to take advantage of investment opportunities. Building portfolios for better investment is a respectable and logical way to participate in the financial market (Cui et al., 2023).

A portfolio is a collection of various financial assets held by an investor. In order to reduce portfolio risk, a number of assets are held in a portfolio for the purpose of diversification. Portfolio optimization is a decision-making process where two or more conflicting objectives, such as maximising return and minimising risks, are considered. The conflicting objectives are solved in three phases: asset selection, asset allocation, and asset management. Asset selection refers to the process of selecting a collection of assets from the same or different asset classes. The process of asset allocation helps investors decide how much money can be invested in which asset(s) to reduce the risk and maximise the return. The final step, asset management, helps the investors evaluate their portfolio and define strategies to buy, sell, or hold an asset (Jothimani et al., 2017).

The portfolio optimization problem is a conspicuous issue in the world of economics (Yaman & Erbay Dalkılıç, 2021) and a persistently popular topic in finance (Gatzert et al., 2021). Building a successful portfolio involves the art and science of balancing trade-offs with the goal of meeting long-term financial goals. To build an optimal portfolio, there are many different strategies, like the Markowitz model portfolio, the Sharpe single index model, and factor models (Senthilkumar et al., 2022). The portfolio design based on the Markowitz model is the most popular. Harry Markowitz laid the groundwork for what is now known as modern portfolio theory in his seminal work (Markowitz, 1952). The main objective is to maximise the expected (mean) return while at the same time minimising the risk (variance) of the portfolio. It relates to the estimation of covariance values, and the cardinality of the model parameters is inversely proportional to the number of assets taken into account. Modern portfolio theory uses correlations as a measure of asset co-movements as input into the portfolio optimization issue (Gupta & Guidi, 2012). This is one of the criticisms of modern portfolio theory. The studies have ignored correlation-based analysis and preferred cointegration tests (Aroskar & Ogden, 2011).

Granger (1981) proposed the concept of cointegration (Hylleberg et al., 1990). Cointegration and correlation are related but different concepts, as a high correlation of returns does not necessarily imply a high cointegration in prices. Correlation reflects co-movements in returns, which are liable to great instabilities over time. It is intrinsically a short-run measure, and correlation-based portfolio management strategies commonly require rebalancing. Cointegration, on the other hand, measures long-run co-movements in prices, which may occur even during periods when static correlations appear low. Hence, asset selection based on the cointegration test may be more effective in long-term investment (Syriopoulos, 2004). In short, cointegration methods are robust to correlation instability (Gallo et al., 2013).

The integration between the investment avenues is a constant challenge for investors (Lee et al., 2019). This will lead to high risk and low return. This condition will change with the construction of efficient portfolios. In addition to identifying the optimal weight for each asset, first of all, we should find the uncorrelated assets for long-term investment (Barbaglia et al., 2016). Constructing an efficient portfolio has always been a puzzling problem (Gunjan & Bhattacharyya, 2022). This study makes an attempt to examine the co-movement between Nifty 50 and ten selected cryptocurrencies in order to find out the efficacy of the cointegration test in enhancing the efficiency of portfolios while considering the result of the cointegration test at the time of portfolio construction. This study adds to the body of literature because there are very few empirical studies based on this concept and no research has been done from the perspective of the Indian stock market.

Literature review

The Johansen cointegration test has been used in numerous research papers to investigate the possibility of links between different financial assets. The majority of studies propose tying the outcomes to investment decision-making. By applying the Johansen cointegration test to examine the interlinkages between the Indian stock market and China, Hong Kong, and the United States (Warne and Suman 2018), the Tokyo Stock Exchange (TSE) and Hong Kong Stock Exchange (HSE) (Singh 2015) discovered that there is no long-run equilibrium relationship between them and concluded that the results benefit better portfolio construction.

Even if the previous studies recommended combining the results of the cointegration test at the time of portfolio construction, studies that empirically examine this are extremely rare. Even though Markowitz's portfolio constructs an efficient frontier, which is the location of the best combination of risk and return (Rabha and Singh 2021), it is needed to rebalance as it provides short-run optimization but not long-term assurance on the investment (Dunis et al., 2011; Meghwani & Thakur, 2018). The cointegration test provides an excellent platform for discovering strategic asset selection as it addresses shortcomings of modern portfolio theory, such as providing long-term investment information, resilience to correlated instability, providing a more robust platform for the design and management of well-diversified portfolios (Gallo et al., 2013), boosting the return of Markowitz portfolios (Dunis et al., 2011), and reducing the risk associated with the Markowitz model portfolio (Naccarato et al., 2021). Contrarily, modern portfolio theory, which relies on short-term information from asset returns and correlation, has the ability to provide knowledge to aid long-term investment decisions, and cointegration, which is primarily based on the concept that first differencing data will result in the loss of trend information and therefore provide incorrect diversification advice, is not justified (Aroskar & Ogden, 2011).

The problem does not end with selecting uncorrelated assets; instead, the optimal weight of selected assets is much more important to increase portfolio efficiency. In India, we found the dominance of naïve investors (Badhani et al., 2022; Kaur & Singh, 2019). In literature, there are more contradictory opinions based on the performance of Naive and optimal asset allocation strategies like efficient portfolios may not be superior to naively diversified portfolios in a statistical sense (Arli et al., 2021); two building methods perform similarly, and naive diversification is exactly as effective as optimal diversification (Platanakis et al., 2018) efficient portfolios are superior to naively diversified portfolios, leading to the conclusion that the optimal portfolio performs better than the naive portfolio (P H & Rishad, 2020). However, the naive-diversification rule should at least be used as a first obvious benchmark when assessing the performance of a specific asset allocation strategy (DeMiguel et al., 2009).

Data and Methodology

Data

The study makes use of the following ten cryptocurrencies: Bitcoin, Ethereum, BNB, XRP, Tether, USD Coin, Cardano, Dogecoin, Polygon, Litecoin, and one stock index, the Nifty 50. Data for the Nifty 50 was collected from investing.com, and data for cryptocurrencies was obtained based on top market capitalization from coinmarketcap.com. Daily data was gathered for three years between April 2019 and March 2022 for the study. Each asset has 743 entries in the data.

Table 1: Investment avenues used in this study

Cryptocurrencies	Stock Index
Bitcoin (BTC)	Nifty 50
Ethereum (ETH)	
BNB	
XRP	
Tether (USDT)	

USD Coin (USDC)
Cardano (ADA)
Dogecoin (DOGE)
Polygon (MATIC)

Litecoin (LTC)

Methodology

The study examines the co-movement between ten selected cryptocurrencies and NIFTY 50. Then construct naïve portfolio and the portfolio based on modern portfolio theory that depicts the efficient frontier of the portfolios with and without considering the result of the cointegration test. That helps to evaluate the potential of the cointegration test for optimizing long-term portfolios. The risk adjusted evaluation of portfolios is also conducted to measure the performance of portfolios.

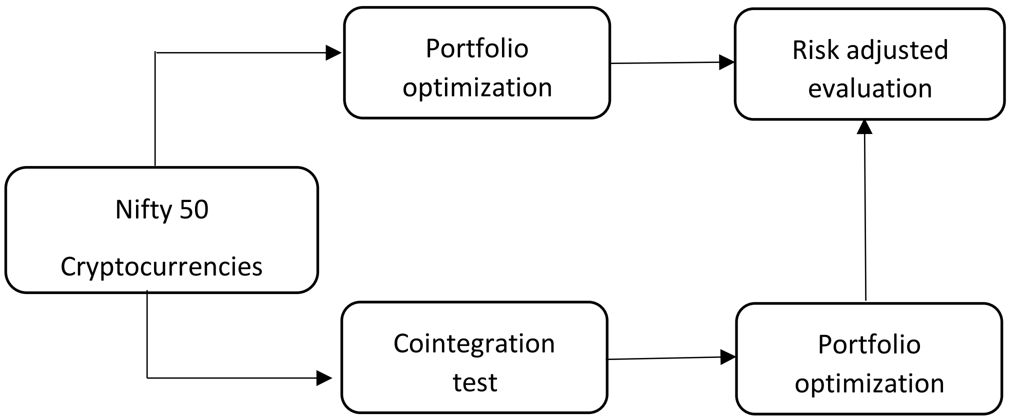

Fig 1: Conceptual frame work

Modern Portfolio Theory

The Markowitz model of portfolio optimization is a popular method for determining the optimum weight of the assets with the highest Sharpe ratio and the lowest return volatility.

The main measure for determining the optimum asset weights is the portfolio's anticipated return. The following is the formula to get the same, as per modern portfolio theory:

$$E(r_p) = \sum_{i=1}^{n} w_i E(r_i) \quad (1)$$

The following formula can be used to determine the variance of a portfolio with more than two assets because return and risk are two sides of the same coin:

$$\sigma_p^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} w_i w_j Cov(r_i, r_j) \quad (2)$$

Using matrix multiplication, choose the best asset weight after obtaining the predicted return.

The portfolio's predicted return is:

$$E(r_p) = W^T R = [w_1 \;\; \dots \;\; w_j] \begin{bmatrix} E(r_1) \\ \vdots \\ E(r_j) \end{bmatrix} \quad (3)$$

R is the vector of predicted returns for each individual asset in the portfolio, and W is the vector of weights for those assets.

The portfolio's variance is determined by:

$$\sigma p^2 = W^T S(W) \quad (4)$$

The following formula is used to compute the portfolio's standard deviation:

$$\sigma_{11} \;\; \dots \;\; \sigma_{1j} \quad w_1 \quad ^{\frac{1}{2}}$$

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$$\sigma_p = \sqrt{W^T S(W)} = \left[[w_1 \; \ldots \; w_j] \begin{bmatrix} \vdots & \ddots & \vdots \\ \sigma j1 & \ldots & \sigma jj \end{bmatrix} \begin{bmatrix} \vdots \\ wj \end{bmatrix} \right] \quad (5)$$

Where S is referred to as the variance-covariance matrix of the covariances between each of the asset returns in the portfolio.

The covariance of an asset's returns for the same asset is the variance of the asset's returns. The definition of W remains the same as above.

Sharpe Ratio

The amount of risk entailed in generating a specific return is measured by a risk-adjusted evaluation. Using a risk-adjusted formula known as the Sharpe ratio, the return on an investment is compared to its risk. The asset weights in a portfolio that produce the highest Sharpe ratio values are the ones to use. To calculate overall risk, the Sharpe's Index uses the standard deviation. In Sharpe's strategy, each portfolio is ranked in accordance with an evaluation metric. Reward and risk premiums are both included in the numerator. Total risk as well as the standard deviation of return are included in the denominator. We will calculate the portfolio's overall risk and return variability with respect to the risk premium.

$$Sp = \frac{E(r_p) - r_f}{\sigma p} \quad (6)$$

$E(r_p)$ stands for Expected return of portfolio where as r_f is represented as risk free rate, at last these are divided with σ_p that is portfolio standard deviation.

Unit root

For the Johansen cointegration test, first we tested for non-stationarity for all indices. The null hypothesis is non-stationarity. A stationary time series requires a consistent mean and variation across time. The idea of stationarity is crucial and required when describing a time series for regression modelling. The Ordinary Least Squares (OLS) calculations will result in spurious regressions with meaningless interpretations if the time series is nonstationary and cointegration is absent. However, the implications of the regression results may be useful if there is at least one stationary combination among the nonstationary time series.

Therefore, a time series must be checked for stationarity before conducting any cointegration analysis. To determine if the series has a stochastic trend or not, we should do a unit root test before running the cointegration test. The stochastic trend was verified using many tests, including the Augmented Dickey-Fuller (ADF), Kwiatkowski-Phillips-Schmidt-Shin (KPSS), Phillips-Perron (PP), etc. This study uses Augmented Dickey-Fuller to test the stationarity of the data series. The augmented Dickey-Fuller (ADF) test is used to count the number of unit roots that each price time series includes. Data in level are first evaluated for stationarity. The first difference form will next be examined if the nonstationary null hypothesis cannot be rejected. Cointegration analysis can be used when test statistics show that all data series integrate in the same order. Therefore, it is important to assess the order of integration of the time series variables. When performing a unit root test, lag length is crucial because it affects how the ACF (autocorrelation function) on the residual is calculated. As a general rule, one should compute the ACF up to one-third to one-quarter the length of the time series. The results are unduly influenced by selecting too little or too much lag. There are numerous lag duration criteria, including LR, AIC, and SC. The study's AIC criteria were used to select the ideal lag length.

Johansen Cointegration Test

A statistical technique called cointegration is used to assist in locating equilibrium or long-run parameters for two or more variables. The Johansen cointegration test finds out whether the independent and dependent variables have a long-term relationship. The choice of the VAR model's number of lags is predicated on the analysis of a cointegration vector between the series, which would then confirm the presence of a linear combination between them. When the series are cointegrated, there is a long-term relationship between them. Even if the series exhibit stochastic tendencies, if the proportions between them are consistent, the series will move together over time and in the same direction (Granger et al., 2000). For this, the series must be stationary with a difference in first order, according to the Stationarity Test. Johansen
(1988) developed a cointegration, based on the rank (r) of the matrix Π and the number of vectors determined by the knowledge of the matrix' rank (r), according to Eq. (7):

$$\Delta X_1 = \delta + \Gamma_1 \Delta X_{t-1} + \cdots \Gamma_{p-1} \Delta X_{t-p+1} + \Pi Y_{t-1} + \quad _{x,t} \tag{7}$$

In this case, $\Pi = \left(\sum_{i=1}^{k} \beta_i\right) - I_g; \Gamma_i = \left(\sum_{j=1}^{i} \beta_j\right) - I_g$, β is the cointegration parameter e ΔX_1 is vector with n variables, $_{x,t} \sim (0, \sum)$ e E($_{x,t}$, $'s$)=0 for t diferent from s. The transformed VAR has g variables on the left of the equation and k-1 lags of the dependent variables on the right, both associated with a coefficient matrix Γ.

There are two likelihood ratios to test for Johansen cointegration, known as the Trace and Max statistics, respectively:

The trace test focuses on testing the null hypothesis, in which the number of distinct cointegration vectors is less than or equal to r or the alternative hypothesis (Engle & Granger, 1987a). In the alternative hypothesis, the number of vectors will be greater than r, according to Eq. (8).

$$\lambda_{\text{trace}}(r) = -T \sum^{g} i{=}v{+}1 \; ln \, [1 - \hat{\lambda}_i] \tag{8}$$

where $\hat{\lambda}_i$ is the estimated value for the ith ordered eigenvalue and r is the number of cointegrating vectors under the null hypothesis. Intuitively, the larger is $\hat{\lambda}_i$, the more large and negative will be ln(1 $-\hat{\lambda}_i$) and hence the larger will be the test statistic. Each eigenvalue will have associated with it a different cointegrating vector, which will be eigenvectors. A significantly non-zero eigenvalue indicates a significant cointegrating vector.

λ_{trace} is a joint test where the null is that the number of cointegrating vectors is less than or equal to r against an unspecified or general alternative that there are more than r. It starts with p eigenvalues, and then successively the largest is removed $\lambda_{trace} = 0$ when all the $\lambda_i = 0$, for i = 1,..., g.

$$\lambda \max (r, r+1) = -T \quad ln \, [1 - \hat{\lambda}_r{+}1] \tag{9}$$

In the maximum eigenvalue test the null hypothesis indicates that the number of vectors is r and consequently, the alternative hypothesis is the existence of r+1 cointegration vectors, represented by Eq. (9). λ_{max} conducts separate tests on each eigenvalue, and has as its null hypothesis that the number of cointegrating vectors is r against an alternative of r + 1.

Error correction model—VECM

The interdependence of the series that is influenced by their order can be examined using the VEC Model. When two variables are cointegrated, they eventually attain long-term equilibrium, and the VEC's estimation allows for defining the speed at which they reach balance. The advantage of VEC is that it retains information about the series level, meaning that the relationships are still there. However, the series must be integrated in the same order, which indicates the need for stationarity tests. If the variables X_{t-i} and Y_{t-i} are integrated in one order I(1) and have cointegration of zero order I(0), the VEC is represented by Eq. (10) (Engle & Granger, 1987):

$$\left.\begin{aligned} \Delta Y_t &= \beta_0 + \beta_i \Delta Y_{t-i} + \beta_j \Delta X_{t-i} + \varphi ECTy,t-i + uy,t \\ \Delta X_t &= \propto_0 + \alpha_i \Delta Y_{t-i} + \alpha_j \Delta X_{t-i} + \varphi ECTx,t-i + ux,t \end{aligned}\right\} \quad (10)$$

where ΔY_{t-i} and ΔX_{t-i} indicate the first difference of the variables lagged i periods; β_0, β_i , β_j , $\propto_0$, α_i, α_j are coeffcients of the model; φ is the long-term coefcient; $u_{y,t}$ and $u_{x,t}$ are terms of random error; ECT_{t-i} are deviations from the long-term balance between ΔY_{t-i} and ΔX_{t-i} lagged i periods. If φ is significant, the short-term variances are adjusted by the long-term cointegration equation errors, enabling us to examine the imbalance proportion of Y_t which is corrected in the subsequent period. The lagged values of the variable ΔX_t can help explain the changes of in another variable ΔY_t , even if the changes in $\Delta Y_t$are irrelevant and considering that the variables are stationary. If $\Delta X_t$and ΔY_t have a common tendency, causality must exist in at least one direction, be it inverse or bi-causality. If the organization of the relations of variances is incorrect, it is necessary to apply the Granger causality test to order the series from the most exogenous to the most endogenous and to obtain the joint signifcance of the endogenous variables lagged in the VEC equation. X^2 indicates the variable's representativity, the greater it is, the more endogenous the p value and the degree of endogeneity.

Results and Discussion

Table 2 provides a detailed summary of the descriptive statistics for the Nifty 50 index and cryptocurrencies used in this study. Among the variables, the Bitcoin exhibits the highest mean value, followed by Nifty 50 and Ethereum, indicating their relatively higher average levels over the study period. In terms of standard deviation, Bitcoin show notably higher value compared to Nifty 50, reflecting the greater volatility and risk associated with cryptocurrencies.

Furthermore, the range, encompassing the minimum and maximum values, demonstrates that cryptocurrencies exhibit a broader variation compared to the Nifty 50, reinforcing their higher susceptibility to price swings. This aligns with the general understanding of the volatile nature of cryptocurrencies. Skewness values indicate that the distribution of most variables is positively skewed, suggesting occasional large upward movements. The kurtosis values reveal that the Nifty 50 (-1.11716), Bitcoin (-0.58736), and Ethereum (-0.80716) exhibit platykurtic distributions, indicating relatively flatter tails and fewer extreme events compared to a normal distribution, while Tether (3.310437) shows a leptokurtic distribution, suggesting heavier tails and a higher likelihood of extreme values.

These descriptive statistics provide a foundation for understanding the behavior and risk-return characteristics of traditional financial indices like the Nifty 50 compared to cryptocurrencies like Bitcoin and Ethereum in the context of this study.

Table 2: Descriptive Statistics

	Nifty 50	*Bitcoin*	*Ethereum*	*Tether*	*BNB*	*USD Coin*	*XRP*	*Cardano*	*Dogecoin*	*Polygon*	*Litecoin*
Mean	13384.42	33211.04	2351.98	1.000194	345.7359	0.999871	0.683738	0.902624	0.118339	0.745487	127.2905
Standard Error	99.25206	469.6893	36.95273	2.14E-05	5.05591	2.54E-05	0.012111	0.028891	0.004738	0.029168	2.59356
Median	12151.15	30286.6	2137.81	1.0002	325	0.9999	0.61438	0.9961	0.059134	0.356	129.8
Mode	17213.6	16831.8	1323.41	1.0002	246.2	0.9999	0.39611	0.106	0.002585	0.02	126.5
Standard Deviation	2705.415	12802.8	1007.258	0.000584	137.8141	0.000692	0.33013	0.787498	0.129162	0.795056	70.69534
Kurtosis	-1.11716	-0.58736	-0.80716	3.310437	0.18984	10.46887	-0.0121	-0.80567	0.888606	-1.13411	0.024057
Skewness	0.28463	0.572593	0.473773	-0.34782	-0.07235	1.093206	0.802673	0.496278	1.026697	0.553838	0.583328
Range	10866.8	51751.7	4222.69	0.0059	645.37	0.0094	1.62494	2.9382	0.685159	2.864	351.389
Minimum	7610.25	15776.2	585.69	0.9968	31.19	0.9965	0.21131	0.027	0.001721	0.01	35.431
Maximum	18477.05	67527.9	4808.38	1.0027	676.56	1.0059	1.83625	2.9652	0.68688	2.874	386.82
Sum	9944626	24675806	1747521	743.1442	256881.7	742.9044	508.017	670.6494	87.92615	553.897	94576.84
Count	743	743	743	743	743	743	743	743	743	743	743

Table 2 provides descriptive statistics of the Nifty 50 and cryptocurrencies used in the study. The highest mean throughout the years has been seen in Nifty 50, Bitcoin and Ethereum.

Table 3: Portfolio optimization without cointegration test.

Summary Statistics	Naïve Portfolio (1/11)	Optimized Sharpe Ratio
MEAN P.A (%)	11.92	14.20
STANDARD DEVIATION P.A (%)	41.11	38.00
SHARPE RATIO	0.117	0.186

Source: Author's Calculation

Table 3 presents the naive and optimized Sharpe ratio findings for the portfolio. The optimized Sharpe ratio shows the portfolio risk and return that have the maximum Sharpe ratio based on modern portfolio theory. It depicts a 14.20% return with a standard deviation of 38.00%. The naïve portfolio allotted equal weights to each asset, and it exhibits inferior performance than the optimized Sharpe ratio. The optimal portfolio yielded 19.12% higher returns and 7.5% lower risks than a naive portfolio. 7.1% is the risk-free rate based on Treasury bills. Due to the market dynamics during the previous three years, both naïve and optimal portfolios resulted in the lowest return and a high standard deviation.

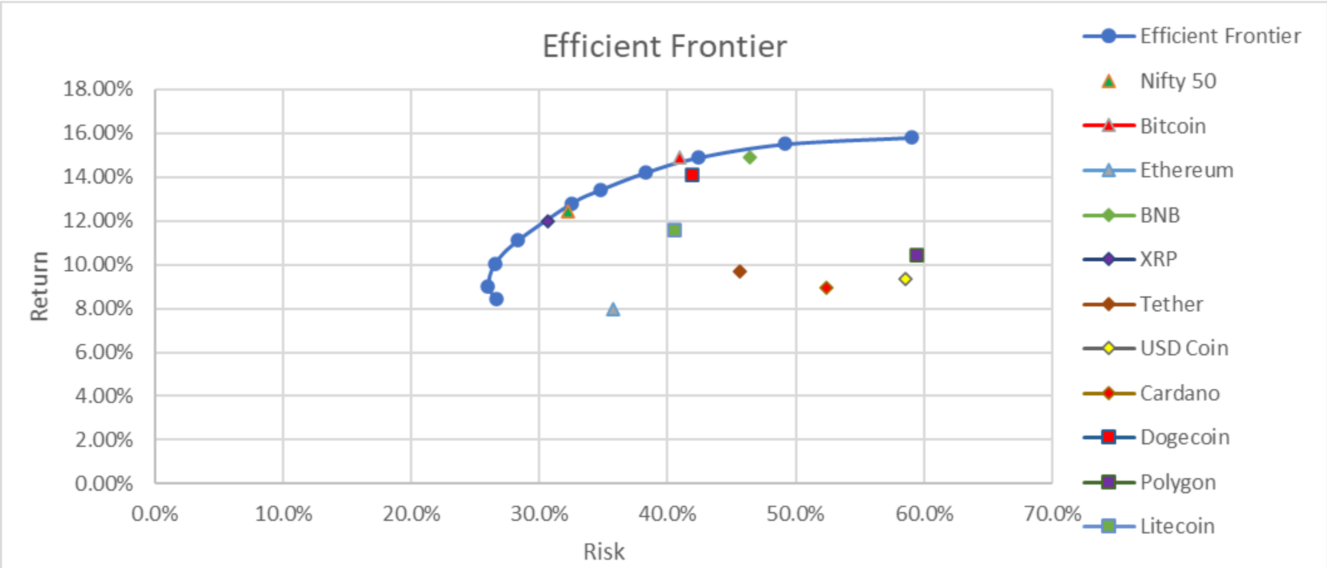

Fig 2: **Efficient frontier without cointegration test**.

The efficient frontier of eleven selected assets is plotted in Fig. 1, along with their risk and return characteristics. The six cryptocurrencies, namely Polygon, USD coin, Cardano, Dogecoin, Bitcoin, and Litecoin, do not fall under the efficient frontier. The rest, including Nifty 50, Tether, XRP, Ethereum, and BNB, fall into the realm of efficiency.

We next examined the efficacy of the cointegration test in enhancing the efficiency of portfolios while considering the result of the cointegration test at the time of portfolio construction. Can the results be improved by using the co-integration technique? Is it possible that omitting investment avenues that move together increases portfolio efficiency because they don't benefit from diversification? The empirical analysis that follows provides answers to these questions.

Table 4: Augmented Dickey Fuller Test

	Levels		First difference	
	With Drift and Trend		With Drift and Trend	
		H0=Non stat		H0=Non stat
Nifty 50	-1.871	Can't Reject H0	-13.501	Reject H0
BTC	-2.010	Can't Reject H0	-4.237	Reject H0
ETH	-3.114	Can't Reject H0	-4.943	Reject H0
BNB	-3.288	Can't Reject H0	-5.112	Reject H0
XRP	-1.561	Can't Reject H0	-7.011	Reject H0
USDT	-2.318	Can't Reject H0	-4.980	Reject H0
USDC	-2.400	Can't Reject H0	-10.994	Reject H0
ADA	-3.251	Can't Reject H0	-7.227	Reject H0
DOGE	-3.218	Can't Reject H0	-12.700	Reject H0
MATIC	-2.318	Can't Reject H0	-11.451	Reject H0
LTC	-3.092	Can't Reject H0	-6.112	Reject H0

Source: Author's Calculation

Table 4 displays the statistics for the Augmented Dickey Fuller test for series with the first difference and at the level. To determine how many unit roots each price time series has, the augmented Dickey-Fuller (ADF) test is used. Initial stationarity testing is done on level data. If the nonstationary null hypothesis cannot be ruled out, we will then investigate the first differenced form. The ADF tests determine if the residual vectors for each asset price series are stationary given the following model parameters.

Table 3 shows that first differencing the series induces stationarity in all assets, suggesting that all our price series are integrated into order 1. Thus, the tau statistic rejects the null hypothesis of non-stationary or the presence of a unit root at the 1% significance level, suggesting that all the price series of assets included in the sample are first-order integrated. Hence, we treat them as I (1) process and proceed with the cointegration analysis.

The level of co-integration between Nifty 50 and each individual asset is displayed in the table below, which tests the null hypothesis of no co-integrating rank.

Table 5: Bi-variate Johansen Cointegration Test

Asset combinations	p	Eigen value	Trace Statistic	Decision
Nifty 50-BTC	P=2	0.000791	3.4233	r=0

Nifty 50-ETH	P=2	0.005011	14.2133	r=1*
Nifty 50-BNB	P=2	0.001732	6.09122	r=0
Nifty 50-XRP	P=2	0.002930	9.50099	r=0
Nifty 50-USDT	P=2	0.003855	15.6628	r=1*
Nifty 50-USDC	P=2	0.005024	20.1100	r=1*
Nifty 50-ADA	P=2	0.002661	10.5004	r=0
Nifty 50-DOGE	P=2	0.004564	17.5578	r=1*
Nifty 50-MATIC	P=2	0.005911	22.3100	r=1*
Nifty 50-LTC	P=2	0.001244	5.04421	r=0

Source: Author's Calculation

We use the bivariate Johansen cointegration test to look into the cointegration results further. It is essential to estimate the optimal number of lags p by using AIC criteria. In this case, the VAR Lag Order Selection Criteria revealed seven lags. The 5% critical value is 15.4947. The results shown above conclusively demonstrate that the Nifty 50 and up to five cryptocurrencies are in long-term equilibrium, namely, Ethereum, Tether, USD Coin, Dogecoin, and Polygon, as the trace statistics of these cryptocurrencies are greater than 15.4947, which is the 5% critical value. Nifty 50 shares one co-integrating rank with each of these cryptocurrencies. Bitcoin has the lowest trace statistic with Nifty 50. Similarly, BNB, XRP, Cardano, and Litecoin could not find strong co-movements.
Two series with the same stochastic trend must have an error-correcting factor that brings them to an equilibrium level. The error correction parameter forces the variables to be closer together for long-term equilibrium. The VECM (vector error correction model) helps to identify shortterm causation among the co-integrated assets and shows how much error is repaired. In practise, it is common to assume that the error correction parameter is statistically significant and negative.

Table 6: VECM for Co-integrated assets

Nifty 50 ETH*	-0.68%*[Nifty 50 (-1)+2004.720-4.54201*ETH(-1)]
Nifty 50 USDT*	-0.22%*[Nifty 50 (-1)+2667.5223-4.139781*USDT(-1)]
Nifty 50 USDC*	-0.95%*[Nifty 50 (-1)-53.9469131-0.06651*USDC(-1)]
Nifty 50 DOGE*	-0.98%*[Nifty 50 (-1)-22.144500-0.21415540*DOGE(-1)]
Nifty 50 MATIC*	-0.70%*[Nifty 50 (-1)-411.5513662-0.0631139*MATIC(-1)]

Source:Author's calculation

Table 6 shows the co-integration equation calculated using VECM between cointegrated assets, which are Nifty 50 and Ethereum, Nifty 50 and Tether, Nifty 50 and USD Coin, Nifty 50 and Dogecoin, and Nifty 50 and Polygon. Equation I in Table 4.9 is a cointegration equation derived from VECM between Nifty 50 and Ethereum. This co-integration equation suggests Ethereum's correct 0.68%, Tether's 0.22%, USD Coin's 0.95%, Dogecoin's 0.98%, and Polygon's 70% disequilibrium with 2 lag variables on a monthly basis.

Below is a demonstration of the system equations of the Nifty 50 and Ethereum:
Nifty 50= D(Nifty 50) = C(1)*(Nifty 50 (-1) -4.54201 *ETH(-1) 2004.720) + C(2)*D(Nifty 50 (-1)) + C(3)*D(Nifty 50 (-2)) + C(4)*D(Nifty 50 (-3)) + C(5)*D(Nifty 50 (-4)) + C(6)*D(ETH(-1)) + C(7)*D(ETH(-2)) + C(8)*D(ETH(-3)) + C(9)*D(ETH(-4)) + C(10)
Use the equation with C(1) as the co-efficient of the co-integrating equation and Nifty 50 as a dependent variable to evaluate long-term causation. In our case, the Nifty 50 is the dependent variable; if the number is proven to be significant, it means that Ethereum (the independent variable) has a long-term effect on it. The Wald test on the co-efficient of lag variables of independent variables, for instance, C(6), which is the lag 1 variable of ethereum, can be used to rule out significant short-run variability. The null hypothesis c(6) = c(7) = c(8) = c(9) = 0 can therefore be evaluated to ascertain whether the variables are causally related in the short run. The study found that each of the cases has short-run volatility.
The multivariate test will examine the co-movement of assets as a group after the bivariate test. If assets move together as a group, diversifying among different assets could not be as profitable. Unrestricted co-integration rank test (trace) would be the preferred option to find out the number of co-integration equations. Table 4.10 below presents

Table 7: Multi-variate Johansen Co-integration Test

Unrestricted Co-integration Rank Test (Trace)				
Hypothesized No. of CE(s)	Eigen Value	Trace Statistic	0.05 Critical Value	Prob.**
None *	0.419059	352.1021	301.1356	0.0000**
At most 1 *	0.338966	249.2890	213.2278	0.0052**

Source:Author's calculation

As per Table 7, as many as two co-integrating equations have been identified. study where ten cryptocurrencies and Nifty 50 are found to be co-integrated in the long run with at most two co-integrating equations. Portfolios formed on the basis of co-integrated groups may not be optimal, and there must exist a greater opportunity if a group can be examined where no cointegration exists.

Multi-variate Johansen Cointegration Test

The preceding bivariate table clearly demonstrates that over the long run, nifty 50 is cointegrated with five assets. In order to construct the portfolio after removing these five cointegrated assets, the multi-variate test conducted after excluding those five assets is explained in the table below:

Table 8: Multi-variate Johansen Cointegration Test with restriction on 5 cryptocurrencies.

Unrestricted Cointegration Rank Test (Trace)				
Hypothesized No. of CE(s)	Eigen Value	Trace Statistic	0.05 Critical Value	Prob.**
None *	0.234559	145.4014	130.5349	0.0709
At most 1 *	0.225662	112.7510	129.6491	0.3933

Source:Author's calculation

The trace statistic is greater than the 5% critical value. Therefore, we fail to reject the null hypothesis that there is no co-integration at the 0.05 level. The portfolio optimization solution in Table 8 has been found with the same investment objective as above:

Table 9: Portfolio Optimization with cointegration test.

Summary Statistics	Naïve Portfolio(1/6)	Optimized Sharpe Ratio
MEAN P.A (%)	12.02	14.90
STANDARD DEVIATION P.A (%)	39.34	30.54
SHARPE RATIO	.1250	.2554

Source: Author's Calculation

Table 9 shows the mean, standard deviation, and Sharpe ratios for both naive and optimized portfolio Sharpe ratios after combining the result of the cointegration test into modern portfolio theory. Ethereum, Tether, USD Coin, Dogecoin, and Polygon are the five that were dropped due to cointegration with the Nifty 50. And finally, portfolios built with the remaining seven assets that are not cointegrated.

When comparing the performance of portfolios before and after the cointegration test, the return improved by 4.9%, while the level of risk declined by 19.6% in the case of an optimum Sharpe ratio. 0.83% more return and 4% lower risk were experienced with the naive portfolio. It shows that when the cointegration test is used in conjunction with modern portfolio theory, the efficiency of portfolios increases. The same conclusion as in Table 3 can be drawn from Table 9. After examining the descriptive statistics of an optimal and naive portfolio, it shows that a naive portfolio has a lower return than an optimal portfolio.

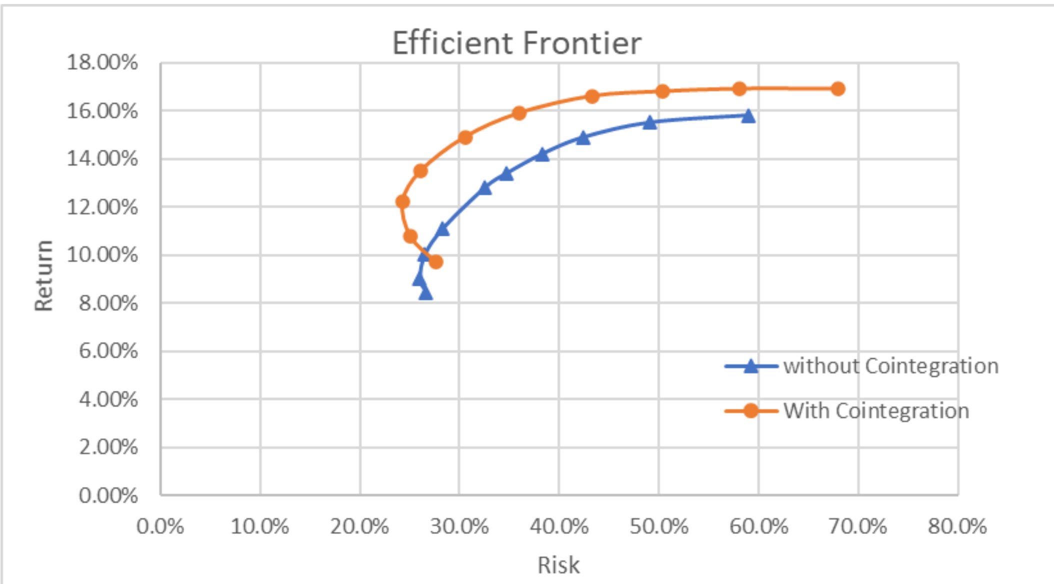

Figure 3: Efficient Frontier with cointegration test.

After combining MPT with co-integration, the efficient frontier moves up the curve (see graph 2), however, it's crucial to note that the frontier is determined using only seven assets combined, as opposed to the original curve's use of eleven assets. We can infer that because the cointegration test yields superior findings, utilising it to design portfolios will be more advantageous to investors. Another benefit that co-integration studies may offer is a reduced sample size, which will unquestionably aid an investor in making an investment decision.

Conclusion

Building a portfolio can be challenging, especially with the range of investment options available today. All financial assets all over the world are currently integrated as a result of globalisation and liberalisation. Whatever happens to one will also have an impact on the others. Investment decisions will be harder in these circumstances. When we apply the greatest techniques to our perplexing issues, we can create the best portfolio. In this study, we can see that applying the cointegration test to modern portfolio theory, one of the most well-known techniques for creating portfolios, will result in a good resolution to our problem. The logic behind applying the cointegration test to modern portfolio theory is that the cointegration test will be helpful in identifying uncorrelated assets over the long term since modern portfolio theory encourages investment in uncorrelated assets such that if certain components of the portfolio decline, others grow.

In the current study, we present empirical proof that there is a better diversification opportunity with cryptocurrencies such as BNB, XRP, Cardano, Litecoin, and Bitcoin because they have no long-run equilibrium with Nifty 50 for the selected time period. When the results of the cointegration test are applied to the portfolio, there is a 37% and a 25.6% increase in the Sharpe ratios of modern and naive portfolios, respectively. In short, when the cointegration test is combined with a modern portfolio, the efficient frontiers boost up with higher return and lower risk, as shown in figures 2 and 3. Additionally, we found that naive portfolios do not overlap with optimal portfolios, but when comparing naive portfolios before and after the cointegration test, it also improves the risk-return level of the portfolios. When rebalancing portfolios with uncorrelated assets, financial advisers, portfolio managers, and investors in India should consider some relevant implications from this study. It will assist investors in thinking rationally about their current portfolio and enhancing its advantages with greater diversification and the selection of least correlated assets. The methodology used in this study can be adopted to any other financial asset, which will be beneficial to investors everywhere in the world.

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