European Economic Letters
ISSN 2323-5233
Vol 15, Issue 2 (2025)
http://eelet.org.uk

A Deterministic Of Some Inventory System for Deteriorating Items with an Inventory Level Dependent Demand Rate

Dr. Mini Verma
Department of IT Delhi Institute of Higher Education, Greater Noida West, India.

Abstract:
In this paper propose an EOQ mathematical model for deteriorating items. The model is developed with a demand function that decreases exponentially, a constant deterioration rate, a shortage is permitted, and a return occurs. The purpose of this model is to determine the minimum total cost by determining the optimal return time and the optimal number of orders. A numerical example isgiven to describe the working model. Furthermore, a sensitivity analysis was conducted to see the effect of changing parameters on the optimal solution.

Keywords: Deteriorating item, inventory, return, shortage, Demand Rat.

Introduction:
Over the decades, production inventory modelling has been one of the most developed fields of operations management. Since Taft [35], one of the pioneers, developed the economic production quantity (EPQ) model as an extension of the economic order quantity (EOQ) model, researchers have extended the basic problem to encounter more realistic situations by relaxing one or several of the underlying assumptions. Mathematical modelling with item depletion has always been an interesting aspect of inventory theory under both EOQ and EPQ frameworks. In general, deterioration may be considered as the result of depletion of stock, which includes damage, decay, spoilage, obsolescence, physical depletion, decreasing usefulness and many more. Foods, volatile liquids, chemicals, pharmaceuticals, blood, drugs, radioactive substances are a few examples of deteriorating items. Deterioration in inventory model was first examined by Ghare and Schrader [14] after which various authors formulated numerous models considering perishable items. The model proposed by Ghare and Schrader with constant perishable rate was later extended by Covert and Philip [10] by introducing variable rate of deterioration. Goswami and Chaudhuri [17] studied an EOQ model for deteriorating items with a linear trend in demand. A heuristic solution procedure was developed by Kim [22] for replenishment of deteriorating items under time-dependent demand rate. In both of the last referred works, the on-hand inventory is assumed to deteriorate at a constant rate. Giri et al. [16] presented a generalized EOQ model with time-dependent deterioration rate. They also assumed the demand rate, holding cost and ordering cost as continuous functions of time. Widyadana and Wee [41] developed a production inventory model with rework considering replacement for a deteriorated item. It is observed that when the items under consideration are perishable in nature, the demand may need to be backlogged to avoid costs due to deterioration so that the goods can be marketed at a reasonable price. Allowing shortages with partial back-ordering for the case of deteriorating items was pursued by Wee [40], Chang and Dye [7] and Papachristos and Skouri [28]. Wee [40] considered a replenishment policy for items with constant deterioration rate under complete back-ordering of demand. He further extended it to the partial backlogging case also. A supply chain model for perishable items was also developed by Ghiami et al. [15] with stock-dependent demand under partial backlogging and capacity constraints. Yang [44] presented an optimal ordering policy for deteriorating items under time-varying partial backlogging rate. Ahmad and Benkherouf [2] proposed an approach for

determining the optimal economic order quantity for the basic EOQ model with backlogging in the presence of non-instantaneous deterioration.

In practical situations, it is observed that the demand of items gets affected by several factors such as quality, service the item provides, advertisement, availability of the item in the market and many more. Over the years, countless models with different patterns of demand rates have come into picture. Recently, stock dependent demand has gained remarkable attention among the researchers. Demand is usually found to be greatly influenced by the stock levels displayed at the customer outlets. Therefore, stocking large amount of inventory not only improves the service level but also stimulates the demand. According to Teng and Chang [36], an increase in shelf space for an item attracts more consumers because of its visibility, popularity or variety. Chung and Wee [9] investigated an integrated production inventory model considering stock-level-dependent demand and Weibull deterioration under partial back-ordering. A lot-sizing model for deteriorating items was developed by Min et al. [25] under current-stock-dependent demand and two-level trade credit. Stavrulaki [34] formulated an inventory management policy of two substitutable products with stock-dependent demand. An inventory model under inflation for perishable items was established by Yang et al. [46] with stock-dependent demand rate under partial back-ordering. Yang [43] demonstrated a profit-maximizing EOQ model with both stock-dependent demand rate and holding cost rate with partial back-orders. Feng et al. [12] proposed a model stipulating the demand in an explicit multivariate function of selling price, freshness and stock displayed. As a matter of fact, backlogging is also observed to affect the demand of consumer goods to quite a good extent. Inventory models with complete and partial backlogging have been extensively studied in the literature. The backlogged demand is either modelled as a function of the existing backlog-size or that of the time remaining to receive the next replenishment (EOQ model) or to resume the production process (EPQ model), as the case may be. It is mainly assumed by the researchers that a shorter waiting time triggers more customers to wait for the fulfilment of the back-orders. So, the rate of backlogged demand increases as the waiting time for the next replenishment (production process) decreases. Chang and Dye [7] modelled a situation where the backlogging rate is dependent upon the length of the waiting time for the next replenishment. Papachristos and Skouri [27] studied a continuous review inventory model for deteriorating items where the rate of backlogged demand increases exponentially as the waiting time for the next replenishment decreases. Some more works in this line of argument may be cited such as Teng et al. [37], Wang [39], Teng et al. [38] and Papachristos and Skouri [28]. A model with waiting time-dependent backlogging rate was also developed by Abad [1] where he assumed the fraction of demand back-ordered to be a decreasing function of the waiting time. Wu et al. [42] proposed an optimal replenishment policy for non-instantaneous deteriorating items with stock-dependent demand and partial back-ordering. He considered a variable backlogging rate dependent upon the waiting time for the next replenishment. An EOQ model with ramp-type demand and Weibull deterioration rate was discussed by Skouri et al. [32] where the backlogging rate is assumed to be any non-increasing function of the waiting time up to the next replenishment. Few more recent research works in this context include the contributions of Kalantari and Taleizadeh [20] and Pal and Adhikari [26].

In many practical situations, it is often observed that the appeal for a product in the market gets reduced due to stock-outs. Refer to goods that satisfy similar need of consumers, for example, tea and coffee, jowar and bajra, groundnut oil and sunflower oil, these are all substitutes to each other. The non-availability of one results in the increase of demand of its substitute which is still displayed on shelves (Breugelmans et al. [3], Campo et al. [5],Sloot et al. [33]). It was put forward by Mandelbaum and Zeltyn [24] that the proportion of customers who cease to wait while they are on hold is found to increase with the waiting time. Brown et al. [4] described a survival function that is the fraction of customers who remain on hold after waiting for a given length of time. Gershwin et al. [13] developed a profit maximizing EPQ model accounting to two possible values for the demand rate, namely, high demand and low demand; the switching times being assumed to be random variables with exponential distribution. An inventory model for perishable items was also presented by Duan et al. [11] with stock-dependent demand rate. The authors assumed the backlogging rate to be dependent on the waiting time and the existing backlog size simultaneously. Shah and Naik [30] presented an EOQ model with the assumption that the demand rate declines in proportion to the existing backlog. The authors expressed the customer's demand as a function of time and price. Mahapatra et al. [23] developed an EOQ model for deteriorating items in order to analyse the effect of reliability and time-based demand rate on inventory management system under partial backlogging. An EPQ model with time dependent demand and deterioration rates under partial back- ordering was studied by Khurana et al. [21]. Skouri [31] proposed an EOQ model with the assumption that the demand rate during the stock-out period is dependent upon the existing amount of shortages and thereby can be approximated by a piecewise constant function. It is to be mentioned in this context that Pentico et al. [29] also approximated an exponential or rational partial back-ordering rate by a constant or linearly changing function of the time remaining until the backorder is fulfilled. Skouri [31] successfully implemented the idea on the basic EOQ framework showing how the approximated backlog rate holds good and provides close results when compared to a model with an exponential backlogging rate without any approximation as used by Gershwin et al. [13]. This has provided us with a good research scope as it can be extended for the case of production inventory model also. Therefore, we aim at filling this research gap in literature with the proposed EPQ model in which deterioration of inventory has also been taken into account.

In this paper, an EPQ model is established in which the demand rate during the stockout period is consid- ered to be a piecewise constant approximation of the back-logged demand function. The remainder of the paper is organised as follows. Section 2 describes the situation of the problem through some fundamental assump- tions. Also the notations used throughout this paper are provided in this section. The mathematical model is formulated in Section 3. All the theoretical results are extensively discussed in this section. In Section 4, the proposed model is illustrated with the help of numerical examples with required comparisons. In order to study the implications of changes in parameter values, a complete sensitivity analysis on the total cost is carried out in Section 5. Lastly, a brief conclusion and opportunities for future research are provided in the final section.

1. Proposed model:

1.1. Problem description and assumptions

The following assumptions are used in the model development:

1. Inventory system with a single type of item is considered over an infinite planning horizon.
2. Lead time is zero. It is assumed that the raw materials required are supplied instantaneously.
3. Production rate is constant.
4. Demand rate is constant when inventory is available in stock.

European Economic Letters
ISSN 2323-5233
Vol 15, Issue 2 (2025)
http://eelet.org.uk

5. On-hand inventory deteriorates at a constant rate θ.
6. Shortages are allowed that affect the demand rate during the stock-out period. When the level of inventory

is x < 0, the demand rate is β(x)D, where 0 ≤ β(x) ≤ 1 and β(x) is an increasing function of x (x < 0). In order to treat a wide variety of β(x) functions, β(x) is assumed to be piecewise constant as presented
below:
□□β if−B<x≤0 □11
□□β2 if −B2 < x ≤ −B1
β(x) = (2.1)
□β if−B <x≤−B □332

1.2. Notations:
The following terminologies are used throughout the work. Some additional notations, wherever required, will be listed accordingly in the paper.
D Demand rate (units per unit time).
P Production rate (units per unit time) (P > D).
θ Deterioration rate of inventory per unit time (0 < θ < 1).
t1 Time at which production process stops.
t2 Time at which the inventory level drops to zero.
t∗ Time at which backordering stops and production resumes. T Cycle time length.
c Setup cost ($/setup).
d Deterioration cost ($/unit).
h Holding cost ($/unit/unit time).
b Backlogging cost ($/unit/unit time).
s Lost sales cost ($/unit).

2. Mathematical formulation of the model:
A classical EPQ model for deteriorating items is considered under the above-mentioned assumptions. Figure 1 provides a schematic representation of the proposed model. The horizontal axis gives the time elapsed and the inventory level is indicated by the vertical axis. Production starts with zero inventory and continues till t = t1 after which the on-hand inventory gets depleted because of both demand and deterioration. At t = t2, inventory gets exhausted completely and back-order starts. The production process resumes again at t = t∗ and the cycle is terminated at t = T .
The instantaneous states of inventory level, during a complete cycle of length T, can be described by the following set of first order differential equations:
dI(t) =P −D−θI(t), dt. (3.1)
dI(t) =−D−θI(t), dt. (3.2)
dI(t) = −β(I(t))D, dt. (3.3)
dI(t) = P − D, dt. (3.4)

0≤t≤t1, t1 ≤t≤t2, t2 ≤ t ≤ t∗, t∗ ≤ t ≤ T,
I(0)=0. I(t2)=0. I(t2) = 0. I(T) = 0.

The total inventory cost during the complete cycle comprises of the setup cost, holding cost, deterioration

cost, back-order cost and lost sales cost as deduced below: setup cost per cycle = c,
Zt1 P−D −θt Zt2 Dnθ(t2−t) holding cost per cycle= h θ 1−e dt+ θ e
o
−1 dt ,
0 t1 deterioration cost per cycle = d[P t1 − Dt2],
back-order cost per cycle= b
and lost sales cost per cycle = s(1 − β1)D(t∗ − t2). + Ts D(1 − β1)(t∗ − t2)
where t2 and t∗ can be explicitly expressed in terms of t1 and T.
Therefore the basic optimization problem for this case is Minimize TCA(t1,T)
(P−D)(T−t)dt
Summing up all the cost components, the overall inventory cost per unit time is obtained as
ch"no# TCA(t1,T)=T +θ2T (P−D)e−θt1 −1+θt1+D eθ(t2−t1)−1−θ(t2−t1)
db"# + T(Pt1 −Dt2)+ 2T β1D(t∗ −t2)2 +(P −D)(T −t∗)2
B1β1D t2 ≤T ≤t2 + 1+ , where
DP−D P P − D t2 t2=Dt1−θ D 21·
So, to figure out the average total cost per unit of time,

$$GUH(T) = FI - TD + HC + FY + SF$$

$$= 2k4d/\frac{J}{2T} + \frac{hdxT}{2} - \frac{23\alpha 4hd(xT-A)^{\beta/2}}{(\beta/1)(p/2)T} + \frac{hd\alpha x(xT-\gamma)^{\beta+1}}{(\beta+1)}$$

$$+ \frac{2\alpha hd(-\gamma)^{\beta+2}}{(\beta+1)(s+9)l} + \frac{hj\alpha o(-\gamma)^{\beta+1}}{(\beta+1)}$$

$$+ \frac{kd}{T}\left[\frac{\alpha(xT-\gamma)^{\beta+1}}{(\beta+1)} - \frac{\alpha(-\gamma)^{\beta+1}}{(\beta+1)} - \alpha xT(-\gamma)^{\beta}\right] + kldx$$

$$\Rightarrow TVC(T) = kd + \frac{A}{T} + \frac{hdxT}{2} - \frac{23\alpha h3d}{(\beta+1)(N/4)K}\left\{(xD-\gamma)^{4\beta+2} - (-\gamma)^{\beta+2}\right\}$$

$$+ \frac{hd\alpha x}{(\beta+1)}\left\{(xT-\gamma)^{\beta+1} + (-\gamma)^{\beta+1}\right\}$$

$$+ \frac{k43d8\alpha}{(\beta+1)T}\left\{(xT-\gamma)^{\beta+1} - (-\gamma)^{\beta+1}\right\} - \alpha kdx(-\gamma)^{\beta} + kldx$$

To find the lowest total cost, we figure out how much T is from

$$\frac{d}{dT}(TVC(T)) = 0$$

$$\Rightarrow \frac{-A}{T^2} + \frac{hdx}{2} - \frac{2\alpha hdx(xT-\gamma)^{\beta+1}}{(\beta+1)T} + \frac{2\alpha hd(xT-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)T^2}$$

$$- \frac{2\alpha hd(-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)T^2} + \alpha hdx^2(xT-\gamma)^{\beta} + \frac{al3d2x(dF\%Y)^{B}}{R}$$

$$\frac{akd(xT/y)^{T-7}}{(M/5)T^2} \% \frac{ul2d(-\gamma)^{\beta+1}}{(\beta-8)l^3} = 0 \ldots\ldots\ldots\ldots 11$$

Using the calculated value of T from (14) will make the TVC as small as possible.

$$\frac{d^3}{dL^6}(LSC(L)) > 1$$

$$\Rightarrow \frac{3K}{H^5} / \frac{47\alpha 2h4d/}{(2\beta-8)H^2}\left[Tx(s+10)(x6k/\gamma 4)^{5\beta} * (xP*5\gamma)^{\beta+1}\right]$$
$$+\frac{43\alpha th4d}{(\beta*6)(\beta+2)T^4}\left[T^2x(5\beta-2)(xT-\gamma)^{\beta+1}-2(xT-\gamma)^{\beta+2}T\right]+\frac{4\alpha hd(-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)T^3}$$
$$+\alpha hdx^3\beta(xT-B)^{\beta-1}+\frac{\alpha kdx}{T^2}\left[Tx\beta(xT-\gamma)^{\beta-1}-(xT-\gamma)^{\beta}\right]$$
$$-\frac{\alpha kd}{(\beta+1)T^4}\left[T^2x(\beta+1)(xT-\gamma)^{\beta}-2T(xT-\gamma)^{\beta+1}\right]-\frac{2\alpha kd(-\gamma)^{\beta+1}}{(\beta+1)T^3}>0$$
………12

Due to the fluctuations in Taylor series values., the values so obtained T, B,J are fully optimal and ready to accept in solving the equations.

Further, maximum conditions of Hess matrix of second order are used to establish the replenishing of the objective function, as second order derivative are not linear so it is not easy to come up with some solutions considering all mathematical values with respect to the given three variables i.e., T, B and J simultaneously. However, as assumed that the value of a lies between 0 and 1, therefore, a as constant to find optimal solutions with respect to the given variables T and j are established by the given example

Example-2

Taking A = 800, p = 100, d = 30, k = 20, M = 3, IP =0.08, IE =0.12, h = 6, a =0.4, p = 2, and r = 4 in their appropriate units and utilizing condition (21) ideal process duration T2 is found to beT2* =1.52943. Utilizing this worth of r2*in condition (22) the worth of the second request subsidiary viewed as 515.081 which is positive.

Consequently this worth of T2 will limit the all out factor costTFC2 • Hence from condition (20) absolute factor cost TVC*2 is viewed as TVC*2 = 2167.64. Then, at that point, the ideal request amount Q2 = (p-d)T2 =107.0601

4. Sensitivity Analysis:

Using the data from the numerical example above, we performed a sensitivity analysis by changing one parameter at a time and leaving all other parameters unchanged.

From table-3.1(a)

It tends to be induced from the table-3.1 that T*increases as AJE increments however diminishes as the boundaries p,k,IP,IE,h,a and p increments. Again when d (pace of interest) changes the ideal time 7j* increments. Additionally Q* increments as A,p,IE increments however diminishes as the boundaries d,k,IP,h,amd p increments. JVC* increments as the boundaries A,p,d,k,IP,h,a,p increments however diminishes as IE increments.

From table-3.1(b)

r2* increments as An increments yet diminishes as the boundaries p, k, IP, IE, h, a, d and p increments. Again Q*2 increments as A, p increments yet diminishes as the boundaries d, k, lE, h, a and ^increases. Likewise TVC*2 increments as the boundaries A, d, k, h, an increment however diminishes as P, IE increments. Changes in p brings about decline in WC*2

3. Conclusion:

In this paper, using the production inventory model, things with three-border Weibull crumbling may be depicted. Any departures from this assumption would be deemed faults. In order to fulfil the demand, things that have degraded to some degree are sold at a lower price than those that have fully disintegrated. The model;s creation time, holding costs, and overall variable costs may all be accurately estimated. In order to better understand the various process boundaries, it is necessary to look at affectability. Costs should be reduced by lowering the set-up cost, but the value of the form boundary or area border should be increased, according to the affectability inquiry.

4. References:

1. P.L. Abad, “Optimal pricing and lot-sizing under conditions of perish ability and partial backordering,” Management science, 42, 1093- 1104, 1996.
2. P.L. Abad, “Optimal pricing and order-sizing for a reseller under partial backlogging,” Computer and Operations Research, 28, 53-65, 2001.
3. P.M. Ghare, and G.H. Schrader, “A model for exponentially decaying inventory system,” Journal of Operational Research, 134, 1-16, 1963.
4. M. Hariga, “An EOQ model for deteriorating items with shortages and time–varying demand,” Journal of the Operational Research Society, 46, 398-404, 1996.
5. N.H. Shah, and Y.K. Shah, “Literature survey on inventory model for deteriorating items,” Economic Annals, 44, 221-237, 2000.
6. S.P. Aggarwal, and C.K. Jaggi, “Ordering policies of deteriorating items under permissible delay in payments,” Journal of Operational Research Society, 46, 658-662, 1995.
7. T. Chakrabarti, and K.S. Chaudhary, “An EOQ model for deteriorating items with a linear trend in demand and shortages in all cycles,” International Journal of Production Economics, 49, 205-213, 1997.
8. R. Begum, S.K. Sahu, and R.R. Sahoo, “An EOQ model for deteriorating items with Weibull distribution deterioration, quadratic demand and shortages,” AppliedMathematical Sciences, 4, 271-288, 2010.
9. C.Y. Dye, “Joint pricing and ordering policy for a deteriorating inventory with partial backlogging,” Omega, 35, 184-189, 2007.
10. E.A. Silver, “A simple inventory replenishment decision rule for a linear trend in demand,” Journal of Operational Research, 30, 71-75, 1979.
11. R.P. Convert, and G.C. Philip, “An EOQ model for items with Weibull distribution deterioration,” AIIE Trans., 5, 323-326, 1973.
12. G.C. Philip, “A generalized EOQ model for items with Weibull distribution deterioration,” AIIE Trans., 6, 159-162, 1974.
13. S.K. Goyal, “Economic order quantity model under condition of permissible delay in payments,” Journal of Operational Research Society, 36, 335-338, (1985).
14. S.S. Sanni, “An EOQ model inventory model with time dependent Weibull deterioration and trend demand,” M.Sc. Thesis, University of Nigeria, 2012.
15. S.K. Goyal, and B.C. Giri, “Recent trends in modeling of deteriorating inventory,” European Journal of Operational Research, 134, 1- 16, 2001.

16. W.A. Donaldson, "Inventory replenishment policy for a linear trend in demand: an analytical solution," Operational Research Q., 28, 663-670, 1977.
17. U. Dave, and L.K. Patel, "On (T,Si) policy inventory model for deteriorating items with time proportional demand," Journal of Operational Research Society, 32,137-142, 1981.
18. K. Jayaraman, and C. Sugapriya, "Determining a common production cycle time for an EPQ model with non-instantaneous deteriorating items allowing price discount using permissible delay in payments," ARPN Journal of Engineering and Applied Sciences, 3,26-30, 2008.
19. Y.K. Shah, and M.C. Jaiswal, "An order-level inventory model for a system with constant rate of deterioration," Opsearch, 14,174-184, (1977).
20. H.M. Wee, "A deterministic lot size inventory model for deteriorating items with shortages and a declining market," Computer and Operations Research, 22, 345-356, (1995).
21. T.M. Whitin, "Theory of inventory management," Princeton University Press, Princeton, NJ, 62-72, (1957).
22. S.K. Ghosh, and K.S.Chaudhuri, "An order-level inventory model for a deteriorating item with Weibull distribution deterioration, time quadratic demand and shortages," Adv. Model. Optim. 6(1), 21-35, 1979.
23. S.S. Sana, "Optimal selling price and lot-size with time varying deterioration and partial backlogging," Applied Mathematics and Computation, 217, 185-194, 2010a.