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# Informational efficiency and modeling of international financial markets

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**Abstract:**
This study aimed to model the informational efficiency and volatility dynamics of international financial markets by analyzing the daily returns of 13 major stock indices (e.g., NASDAQ, Euro Stoxx 50, FTSE 100) from 2017 to 2025, narrowing the sample to five least-correlated indices for diversification. Using econometric tools such as ARMA, ARCH, GARCH, EGARCH, and TARCH models, alongside unit root tests (ADF, PP, KPSS) and volatility clustering diagnostics, the study confirmed market efficiency through a White Noise process in mean returns, while identifying significant volatility clustering, asymmetry, and leptokurtosis. Results showed that EGARCH(1,1,1) models best captured asymmetric volatility effects for most indices, with forecasts projecting the NASDAQ Composite to yield the highest return (14.06%) and risk (1.62%) by June 2025.

**Keywords**: Financial markets, volatility clustering, ARCH models, market efficiency, asymmetric returns, forecasting.

**JEL Code : G1 ,C5 ,G14 ,C53.**

## 1. Introduction

The hypothesis of informational efficiency in financial markets is one of the main pillars of modern finance, particularly in guiding investors' allocation decisions. The concept of market efficiency can be interpreted in various ways depending on the component considered, but generally, a financial market is deemed efficient when the price of a financial asset incorporates, at every moment, all available information and reflects the underlying economic values. Thus, in a financially efficient market from an informational perspective, searching for additional information is futile since all information is already embedded in the asset's price. In such a case, knowledge of historical series capturing the realization of an asset's price or return could not be used to achieve abnormally improved returns.

In statistical terms, the informational efficiency of a financial market implies that an asset's return follows a stochastic process. Models of the ARMA (AutoRegressive Moving Average) class enable the modeling of many such processes based on a single realization. Moreover, the conditional volatility of a return series, measured by its variance, has no reason to be constant, stationary, or linear, as assumed in classical ARMA models. Consequently, new econometric models have been proposed, notably the ARCH (AutoRegressive Conditional Heteroscedasticity) class and its derivatives.

These models aim, on the one hand, to account for two commonly observed phenomena in financial asset returns: the alternation of periods of low volatility and periods of high volatility, as well as the presence of high leptokurtosis. On the other hand, seeking to explore second-order market inefficiencies and the potential predictability of returns.

The objective of this article is to explore the investment opportunities available to investors in the context of international diversification. In this approach, investment opportunities are provided by the main international financial markets, represented by the returns of their corresponding financial indices. Constructing investment portfolios requires prior possession and knowledge of forecasts (future values) of the various returns and associated risks. Under these conditions, ARMA-ARCH statistical models come into play to provide consistent forecasts.

**The general problem can be formulated as follows:**
Is it feasible to model and forecast the returns and risks of financial indices under the hypothesis of informational efficiency of the corresponding financial markets?
The modeling approach proceeds through three complementary general steps. From the perspective of international diversification, in the first step, selecting from the set of international financial markets those considered significant and representative of a varied and balanced international diversification. Then, still in the first step, it will be retained from this set the financial markets that are least integrated, where the level of integration is measured by the historical correlation coefficients between the values of their corresponding stock indices.

In the second step, based on the hypothesis that the returns of the stock indices of the selected financial markets represent, on average, the returns of all constituent financial assets, seeking at least a second-order inefficiency to specify and model the returns of the selected financial markets (potentially using ARCH-class models and their derivatives). In the third and final step, appropriate models will be used to make short-term forecasts of expected averages and corresponding risks. Finally, noting that the results obtained can serve as a solid foundation for constructing investment portfolios in the context of international diversification.

## 1. Selection of Financial Markets and Descriptive Analysis of Returns:

In this section, proceeding with the selection of financial markets that are potential targets for investment and international diversification. However, a presentation of the data, a descriptive analysis of the return series, and the determination of their basic statistical characteristics are necessary before any further analysis.

### 1.1. Presentation of the Data[1]:

From the wide variety of financial markets available in the global financial sphere, 13 financial markets have been selected, represented by stock indices, which considered the most significant and which meet the criteria for effective international diversification. These criteria include ensuring geographic diversification, currency diversification, and diversification across financial markets of countries with different levels of development. The calculation method for their representative indices ensures continuity in their values following events such as stock splits, consolidations, or changes in the list of companies constituting the index.

**The selected stock indices are:**

• **Dow Jones Industrial Average**: A U.S. index of the New York stock exchanges, it has represented, since 2018, the stock prices of the 30 most significant companies.

• **Nasdaq Composite***: A stock index that measures the performance of companies listed and traded on the NASDAQ stock exchange, the second-largest equity market in the United States by trading volume.

• **S&P 500**: A stock index based on 500 large companies listed on U.S. exchanges. It covers approximately 80% of the U.S. stock market by capitalization.

• **Russell 2000**: A U.S. stock index, its 2,000 constituent stocks represent small companies from various sectors and industries present in the main U.S. financial markets.

• **Euro Stoxx 50**: A stock index for the Eurozone, comprising 50 components corresponding to the 50 largest companies by market capitalization.

• **FTSE 100**: A stock index of the 100 highest-capitalized British companies listed on the London Stock Exchange, representing over 70% of the London financial market's capitalization.

• **DAX**: The main German stock index, its value is based on the stock prices of the 30 most significant companies listed on the Frankfurt Stock Exchange.

• **CAC 40**: The main stock index of the Paris Stock Exchange, its value is determined based on the prices of 40 listed stocks.

• **Nikkei 225**: The main Japanese stock index of the Tokyo Stock Exchange, composed of the stocks of 225 companies.

• **Shanghai Composite**: The stock index of the Shanghai Stock Exchange, it is the most widely used index to reflect the performance of the Chinese financial market.

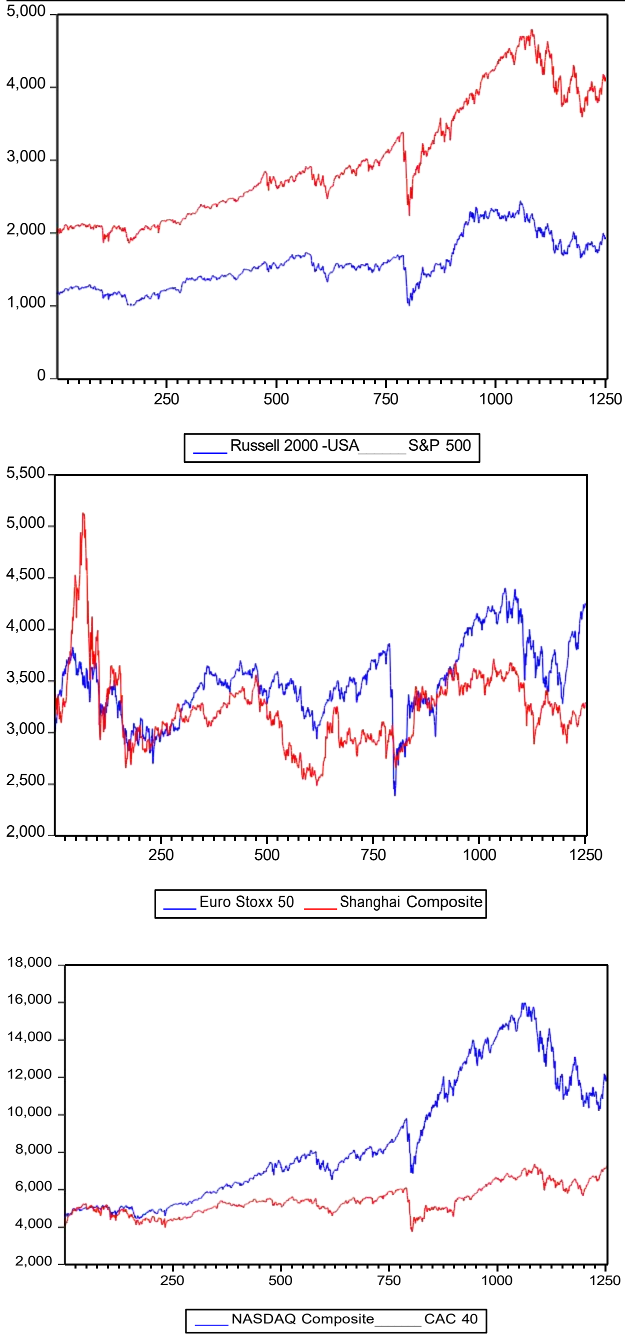
• **Nifty 50**: The main stock index of India, its value is determined based on the stock prices of 50 large companies listed on the National Stock Exchange of India.

• **Tadawul All Share**: The index of the Saudi Arabian stock exchange, its value is composed of the stocks of over 100 companies. It represents nearly 75% of the trading volume of equity markets in the Gulf Cooperation Council.

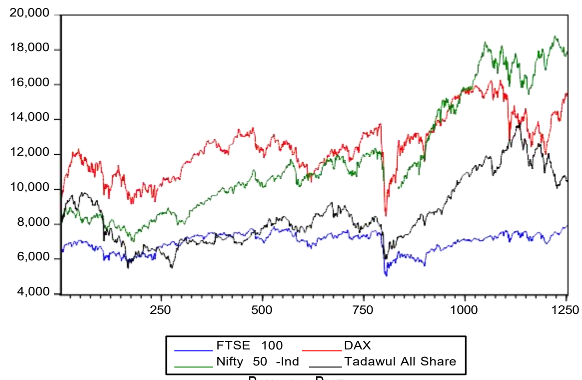
• **Bovespa**: The most significant index of the São Paulo Stock Exchange in Brazil, composed of approximately 100 of the country's largest market capitalizations.

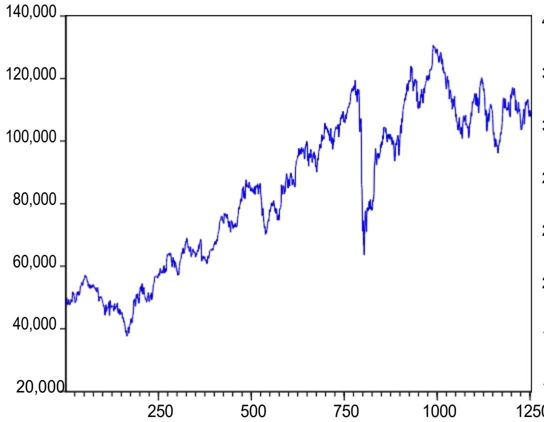
The values of these stock indices are the daily closing values covering the period from January 6[2], 2017, to February 16, 2025, resulting in a sample of 1,254 daily observations (Charts °01). In fact, the use of high-frequency daily data is necessary to detect the potential presence of the clustering phenomenon.

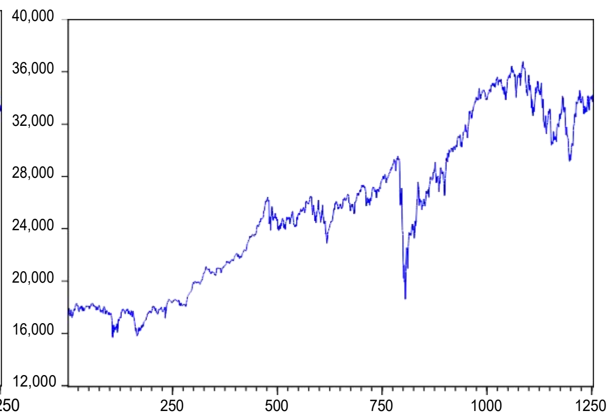
**Chart No. 01: Evolution of the main stock market indices from 01/01/2017 to 02/16/2025**




2 Only the days common between the different indices were taken.




**Sources :**[https://fr.investing.com](https://fr.investing.com)

### 1.1. Selection of Stock Indices:

The analysis of the elements of the historical correlation matrix (Table No. 01 below) reveals that nine indices namely, Dow Jones Industrial Average, Nasdaq Composite, S&P 500, Russell 2000, DAX, CAC 40, Nikkei 225, Nifty 50, and Bovespan. Exhibit similar historical trends. This similarity is reflected through positive correlation coefficients close to 1 (> 0.8). In the context of portfolio management and according to Markowitz's portfolio management theory, diversifying investments across indices that are strongly positively correlated does not significantly improve the portfolio's expected return nor reduce the associated risk. Therefore, it is more effective to invest in a single representative asset (index). To enhance the efficiency of diversification, the Nasdaq Composite was selected to represent the group of nine indices, as it is, on average, the least correlated with the remaining four indices (Table No. 01). Consequently, five indices were retained for further analysis: Nasdaq Composite, Euro Stoxx 50, FTSE 100, Shanghai Composite, and Tadawul All Share.

**Table No. 01: Correlation matrix between the 13 stock market indices**

| | DOW_JONES_INDUSTRIAL | NASDAQ_COMPOSITE | S_P_500 | RUSSELL_2000__USA | EURO_STOXX_50 | FTSE_100 | DAX | CAC_40 | NIKKEI_225__ | SHANGHAI_COMPOSITE | NIFTY_50_IND | BOVESPA__BRAZ | TADAWUL_ALL_SHARE |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|

| | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **DOW_JONES_INDUSTRIAL_AVERAGE_** | 1,0000 | 0,9580 | 0,9828 | 0,9442 | 0,7506 | 0,4023 | 0,9039 | 0,9115 | 0,9463 | 0,0300 | 0,9508 | 0,9417 | 0,7700 |
| **NASDAQ_COMPOSITE** | 0,9580 | 1,0000 | 0,9857 | 0,9485 | 0,7348 | 0,2039 | 0,8907 | 0,8513 | 0,9398 | 0,1577 | 0,9142 | 0,8849 | 0,7618 |
| **S_P_500** | 0,9828 | 0,9857 | 1,0000 | 0,9447 | 0,7651 | 0,3145 | 0,9031 | 0,9081 | 0,9470 | 0,1149 | 0,9600 | 0,8974 | 0,8124 |
| **RUSSELL_2000_USA** | 0,9442 | 0,9485 | 0,9447 | 1,0000 | 0,7991 | 0,3908 | 0,9248 | 0,8832 | 0,9484 | 0,1589 | 0,8920 | 0,8667 | 0,7361 |
| **EURO_STOXX_50** | 0,7506 | 0,7348 | 0,7651 | 0,7991 | 1,0000 | 0,5342 | 0,9205 | 0,9205 | 0,8011 | 0,4328 | 0,7563 | 0,6185 | 0,7277 |
| **FTSE_100** | 0,4023 | 0,2039 | 0,3145 | 0,3908 | 0,5342 | 1,0000 | 0,4893 | 0,5838 | 0,3427 | -0,0726 | 0,4062 | 0,3463 | 0,3515 |
| **DAX** | 0,9039 | 0,8907 | 0,9031 | 0,9248 | 0,9205 | 0,4893 | 1,0000 | 0,9324 | 0,9115 | 0,2714 | 0,8552 | 0,8046 | 0,7082 |
| **CAC_40** | 0,9115 | 0,8513 | 0,9081 | 0,8832 | 0,9205 | 0,5838 | 0,9324 | 1,0000 | 0,8974 | 0,1892 | 0,9248 | 0,7943 | 0,8250 |
| **NIKKEI_225__** | 0,9463 | 0,9398 | 0,9470 | 0,9484 | 0,8011 | 0,3427 | 0,9115 | 0,8974 | 1,0000 | 0,2243 | 0,9257 | 0,8665 | 0,7957 |
| **SHANGHAI_COMPOSITE** | 0,0300 | 0,1577 | 0,1149 | 0,1589 | 0,4328 | -0,0726 | 0,2714 | 0,1892 | 0,2243 | 1,0000 | 0,0812 | -0,0538 | 0,3024 |
| **NIFTY_50_IND** | 0,9508 | 0,9142 | 0,9600 | 0,8920 | 0,7563 | 0,4062 | 0,8552 | 0,9248 | 0,9257 | 0,0812 | 1,0000 | 0,8483 | 0,8585 |
| **BOVESPA_BR** | 0,9417 | 0,8849 | 0,8974 | 0,8667 | 0,6185 | 0,3463 | 0,8046 | 0,7943 | 0,8665 | -0,053 | 0,8483 | 1,0000 | 0,6487 |
| **TADAWUL_ALL_SHARE** | 0,7700 | 0,7618 | 0,8124 | 0,7361 | 0,7277 | 0,3515 | 0,7082 | 0,8250 | 0,7957 | 0,3024 | 0,8585 | 0,6487 | 1,0000 |

**Source:**Extraitsd'Eviews 10

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**1.1. Returns and Descriptive Statistics:**

Modern finance employs models in which the time intervals considered are infinitesimal. This approach is more realistic than the discrete-time approach, as it implies that a change in the price of a financial asset can occur at any moment. In continuous time, it is demonstrated that the return of a given stock is expressed as:

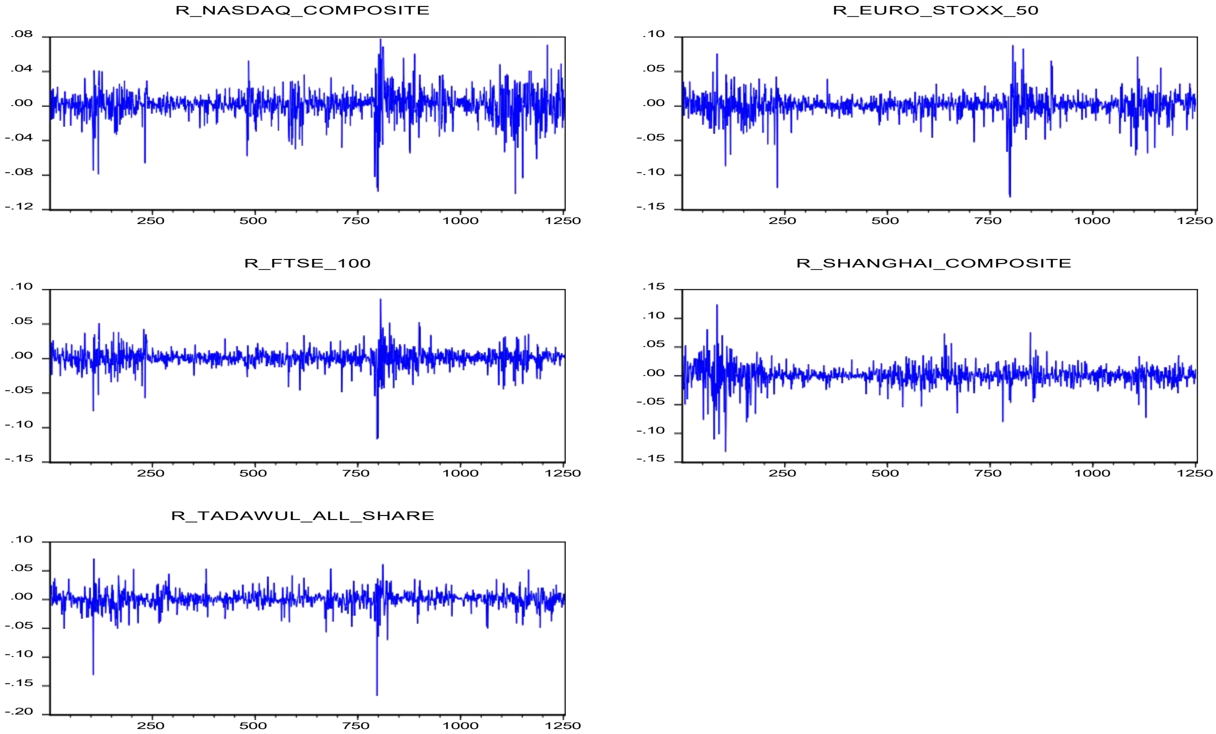
$$R_t = ln\left(\frac{C_t}{C_{t-1}}\right) \ldots \ldots \ldots (1)$$

which is the natural logarithm. It is also shown that the total return, , calculated over a period , is given by the formula:

$$R_T = ln\left(\frac{C_T}{C_{T-1}} * \ldots * \frac{C_t}{C_{t-1}}\right) = ln\left(\frac{C_T}{C_{T-1}}\right) + \cdots + ln\left(\frac{C_t}{C_{t-1}}\right) = \sum_{t=1}^{T} R_t \ldots \ldots \ldots (2)$$

The daily returns of the 5 indices selected, noted respectively R_Nasdaq_Composite, R_Euro_Stoxx 50, R_FTSE_100, R_Shanghai_Composite and R_Tadawul_All_Share, are calculated in continuous time mode according to formula (1), their graphical representations are shown in the following graph n°02.

**Chart No. 02: Evolution of the returns of the 5 indices**




Before any modeling, it is essential to conduct a descriptive study. The corresponding statistics of the return series are summarized in the following Table No. 02:

**Table No. 02: Descriptive statistics of index returns**

| | *R_NASDAQ_ COMPOSITE* | *R_EURO_S TOXX_50* | *R_FTSE_ 100* | *R_SHANGHAI_ COMPOSITE* | *R_TADAWUL_ ALL_SHARE* |
|---|---|---|---|---|---|
| Average | 0,000757 | 0,000285 | 0,000184 | -2,48E-05 | 0,000215 |

| | | | | | |
|---|---|---|---|---|---|
| Maximum | 0,078 | 0,088 | 0,087 | 0,124 | 0,071 |
| Minimum | -0,102 | -0,132 | -0,117 | -0,132 | -0,167 |
| Standard deviation | 1,68% | 1,67% | 1,34% | 1,77% | 1,51% |
| Skewness | -0,863 | -1,188 | -1,148 | -0,669 | -1,847 |
| Kurtosis | 8,815 | 14,601 | 15,512 | 12,300 | 21,601 |
| Obs. | 1253 | 1253 | 1253 | 1253 | 1253 |

**Source:**Extraitsd'Eviews 10

Over the entire period, the average return is nearly zero for all five indices. The SHANGHAI_COMPOSITE index achieved the highest return of 12.40%, while the TADAWUL_ALL_SHARE index recorded the lowest return of -16.75%. By comparing the standard deviation values to the mean values, it is observed that the returns of the five indices exhibit very high variability over the entire period, with an asymmetry toward returns above the mean (Skewness < 0). The Kurtosis coefficient, greater than 3 for all five return series, indicates a high probability of extreme values occurring. A visual examination of the return charts (Chart No. 02 above) reveals the presence of the clustering phenomenon, with varying degrees across the five series, where large (small) variations in returns are often followed by large (small) variations.

### 1.1. Study of the stationarity of yield series:

The theory behind estimating ARMA models is based on the stationarity of the time series under study. The formal method for testing the stationarity of a series is the unit root test. The following Table 03 summarizes the results of the unit root tests: Augmented Dickey-Fuller (ADF), Phillips-Perron (PP), and Kwiatkowski-Phillips-Schmidt-Shin (KPSS) for the five series analyzed:

**Table No. 03: Results of the unit root test**

| | **Trend and constant term.**[3] | | | **Terme constant** | | |
|---|---|---|---|---|---|---|
| | **ADF** | **PP** | **KPSS**[4] | **ADF** | **PP** | **KPSS** |
| *R_NASDAQ_COMPOSITE* | 0,00 | 0,00 | 0,146 | 0,00 | 0,00 | 0,463 |
| *R_EURO_STOXX_50* | 0,00 | 0,00 | 0,146 | 0,00 | 0,00 | 0,463 |
| *R_FTSE_100* | 0,00 | 0,00 | 0,146 | 0,00 | 0,00 | 0,463 |
| *R_SHANGHAI_COMPOSITE* | 0,00 | 0,00 | 0,146 | 0,00 | 0,00 | 0,463 |
| *R_TADAWUL* | 0,00 | 0,00 | 0,146 | 0,00 | 0,00 | 0,463 |

**Source:**Extraitsd'Eviews 10

At the 5% significance level, the values of the three tests indicate the absence of a unit root for all five return series. Concluding that the series are stationary at their level around their means ($I(0)$).

[3]Meaning at 5% level
[4]The assumptions of the KPSS test are reversed compared to the ADF and PP tests.

### 1.2. Model identifications :

The analysis of time series based on observed data involves two key steps: the identification step and the construction of the representative model. These steps require an understanding of the characteristics of the processes in terms of autocorrelation functions (ACF) and partial autocorrelation functions (PACF). The results of the unit root test suggest stationarity in mean, but a visual examination of the graphical representations (Chart No. 02) suggests that they are not stationary in conditional variance.

The examination of the corresponding ACF and PACF functions, represented by correlograms and the Ljung-Box statistic, suggests stationarity of the White Noise type for all five return series (ARIMA(0,0,0) model), with a generating model of the form:

$$R_{it} = c_i + \varepsilon_{it}(i = \overline{1,5}) \dots \dots \dots (3)$$

The estimation results of model (3) for the five return series are summarized in the following Table No. 04:

**Table No. 04: estimation of the models of the 5 yield series**

| | C | Prob. | D.W |
|---|---|---|---|
| *R_NASDAQ_COMPOSITE* | 0,000757 | 0,1123 | 2,11 |
| *R_EURO_STOXX_50* | 0,000285 | 0,5453 | 2,02 |
| *R_FTSE_100* | 0,000184 | 0,6290 | 2,09 |
| *R_SHANGHAI_COMPOSITE* | -0,0000248 | 0,9605 | 1,99 |
| *R_TADAWUL* | 0,000215 | 0,6142 | 1,97 |

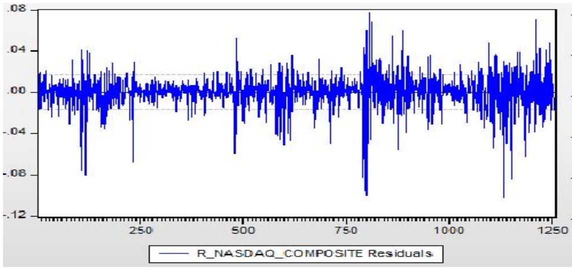
**Source:**Extraitsd'Eviews 10

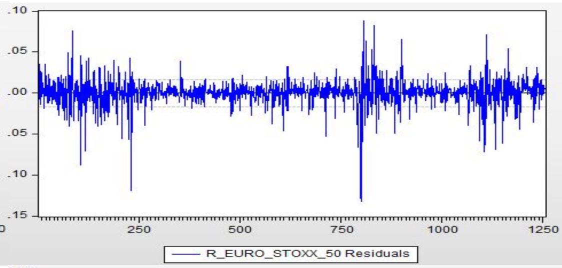
Keeping the constant term (C) in the 5 models, corresponding to the average daily yield, even if it is not statistically significant $(\mathrm{P-value} > 5\%)$. In fact, aggregating daily returns over a compound period (formula (2)) in the forecasting step makes it possibly important and meaningful. The Durbin Watson statistics close to 2 from the 5 models indicate the absence of autocorrelation of errors (of order 1). As the error terms du modèle (1) ne sont $\varepsilon_{it}(i = \overline{1,5})$ than a linear transformation of the yield series, they consequently have the same statistical characteristics reported in tables 02 and 03. Therefore, it is concluded that the errors represent a non-Gaussian White Noise process.

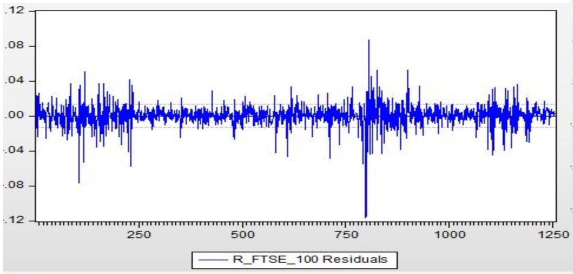
### 1.3. Test for the presence of ARCH effect:

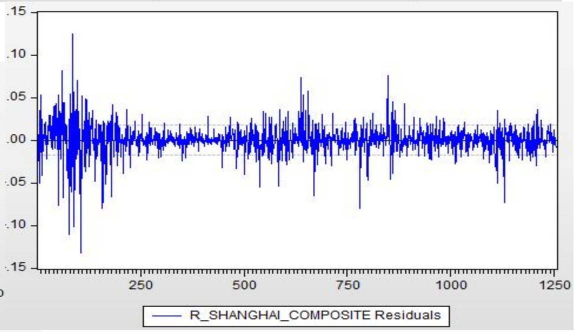
**2.** One of the fundamental assumptions in the previous modeling is that the volatility of the variance of the error term is constant (homoscedastic). In certain applications, particularly in financial series, this assumption does not hold. The visual examination of the residuals from model (3), represented in Chart No. 03 below, reveals variability in the variance for all five models. Observing variances that change over time, where large (respectively small) variations are followed by small (respectively large) variations, a phenomenon known as "clustering or the ARCH effect."

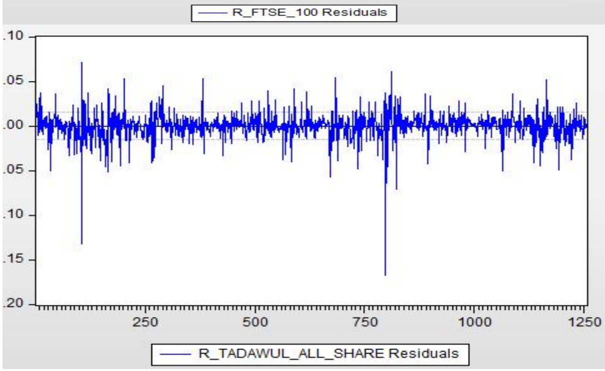
**Graphique n°03 :**résidus de l'estimation du modèle (3)$(ARIMA\ (0,0,0))$




The results of the Lagrange Multiplier (LM) test for the presence of the ARCH effect are summarized in the following table no. 05.

**Table No. 05: Results of the ARCH Effect Presence Test**

| | **L'ordre *q*** | ***LM*** | **P-value** |
|---|---|---|---|
| *R_NASDAQ_COMPOSITE* | 4 | 132,08 | 0,000 |
| *R_EURO_STOXX_50* | 1 | 4,08 | 0,043 |
| *R_FTSE_100* | 2 | 186,38 | 0,000 |
| *R_SHANGHAI_COMPOSITE* | 2 | 56,45 | 0,000 |
| *R_TADAWUL* | 1 | 10,02 | 0,001 |

**Source:**Extraitsd'Eviews 10

The values of the LM test statistics, below the 5% threshold, suggest the rejection of the homoscedasticity hypothesis for the residuals from model (3) and confirm the presence of an ARCH effect for all five return series.

## 2.1. Estimation of ARCH models:

ARCH modeling naturally focuses on eliminating conditional heteroscedasticity, which has been justified thus far from both statistical and theoretical perspectives. In fact, ARCH models and their generalizations account for most characteristics of financial series, such as thick-tailed distributions, the presence of volatility clustering, and nonlinearity.

To this end, estimating, for each of the five return series, the ARCH, GARCH, GARCH-M (M-variance and M-standard deviation), EGARCH, and TARCH models using the Maximum Likelihood method and the Newton-Raphson optimization algorithm. Next, selecting the most appropriate model from the estimated models based on information criteria (Akaike (AIC), Schwarz (SIC), and Hannan-Quinn (HQ)). Then, verifying the normality assumption of the errors from the estimated models by applying the Jarque-Bera test. If the normality of the errors is not confirmed, re-estimating the corresponding model using alternative empirical distributions (Student's t and GED) and select the best model based on the same information criteria. Finally, to validate the models, checking after each estimation for the absence of the ARCH effect (LM test).

### 2.1.1. Estimation of models $ARCH(q)$ :

The general form of the ARCH(q) model is given by the formula:

$$\begin{cases} R_{it} = c_i + \varepsilon_{it} \\ \varepsilon_t = \eta_t \sqrt{h_t} \\ h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \ldots + \alpha_q \varepsilon_{t-q}^2 \end{cases} \quad \ldots\ldots\ldots (4)$$

avec$\eta_t \rightsquigarrow IID\ N(0,1)$

The results of the estimations are summarized in the following table:

**Table No. 06: Model estimation results**

| | **Normalité des Erreurs** | **DEE[5] retenue** | **L'ordre *q*** | **Disparition effet ARCH** |
|---|---|---|---|---|
| *R_NASDAQ_COMPOSITE* | Non | Student | 5 | Oui |
| *R_EURO_STOXX_50* | Non | Student | 5 | Oui |
| *R_FTSE_100* | Non | Student | 5 | Oui |
| *R_SHANGHAI_COMPOSITE* | Non | Student | 5 | Oui |
| *R_TADAWUL* | Non | GED | 1 | Oui |

**Source:**Extraitsd'Eviews 10

The lag order "p" determines the duration of the persistence of the shock to the conditional variance of errors over time; the larger "p" is, the more persistent the volatility. Observing that the lag order is relatively large for the return series (with the exception of the TADAWUL index), indicating a persistence of the shock to the conditional variance of errors over time. Additionally, the Jarque-Bera normality test reveals that the errors from the estimation of the ARCH models for the five return series are not normally distributed. Consequently, selecteing the distribution, between Student's t and GED, that yields the minimum values according to the selection criteria of Akaike, Schwarz, and Hannan-Quinn. Ultimately, the LM test indicates the disappearance of the ARCH effect in the five selected models.

### 2.1.2. Estimation of GARCH (p, q) models::

The rationale for the GARCH model comes from the idea that its form can represent a high-order ARCH process. In this type of model, time-distant information on the conditional variance of errors is taken into account. The general form of the model is given by the formula:

[5] Distribution Empirique des Erreurs

$$\begin{cases} R_{it} = c_i + \varepsilon_{it} \\ \varepsilon_t = \eta_t \sqrt{h_t} \\ h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \ldots + \alpha_q \varepsilon_{t-q}^2 + \beta_1 h_{t-1} + \ldots + \beta_p h_{t-p} \end{cases} \ldots \ldots \ldots (5)$$

avec$\eta_t \leadsto IID\ N(0,1)$ and under constraints:

$$\begin{cases} \alpha_0 > 0 \\ \alpha_i \geq 0\ pour\ i = \overline{1,q} \\ \beta_i \geq 0\ pour\ i = \overline{1,p} \end{cases}$$

The choice of the form of the mean function and the number of lags and for the conditional variance function for the different series led to the following results:

**Table n°07 :** Model estimation results $GARCH(p,q)$

| | Normality of Errors. | DEE retained | The Order | | Disappearance ARCH Effect |
|---|---|---|---|---|---|
| | | | ***p*** | ***q*** | |
| *R_NASDAQ_COMPOSITE* | Non | Student | 1 | 1 | yes |
| *R_EURO_STOXX_50* | Non | Student | 1 | 1 | yes |
| *R_FTSE_100* | Non | Student | 1 | 1 | yes |
| *R_SHANGHAI_COMPOSITE* | Non | Student | 1 | 1 | yes |
| *R_TADAWUL* | Non | Student | 1 | 1 | yes |

**Source:**Extraitsd'Eviews 10

The results indicate that all five return series can be represented by a GARCH(1,1) model with a Student's t empirical distribution for the errors. The sum of the ARCH and GARCH coefficients, close to 1 for all return series, confirms the persistence of the shock on volatility over time.

### 2.1.3. Estimation of IGARCH (p,q) models: :

The model is a specification for processes where the variance function is non-stationary in level. However, the results of the ADF test on the squared residuals from the mean function (3) above for the five return series showed that they are stationary in level, leading us to rule out the specification. This result indicates that a shock to the variance of the errors does not propagate to its future values in an explosive manner; the shock dissipates over time for all five return series.

### 2.1.4. Estimation of ARCH-M models:

It can be considered that the mean and variance of returns move in the same direction. The ARCH and GARCH models with a mean effect take this hypothesis into account, where the expected return is linked to the expected risk (variance or standard deviation).

The ARCH-M model is specified by:

$$R_{it} = X_{it}'B + \delta g(h_t) + \varepsilon_t \ldots \ldots \ldots (6)$$

avec$\varepsilon_t/\psi_{t-1} \leadsto N(0,h_t)$ et $h_t$ est un processus ARCH ou GARCH et $X_{it}'B$ est une constante.

The estimates of the different models for the 5 yield series are summarized in the following table:

**Table No. 08: Model estimation results $GARCH(p,q) - M$**

| | | Function of the mean | | DEE restraint | Variance function | | Disappearance effect ARCH[6] |
|---|---|---|---|---|---|---|---|
| | | Signe | Sig. | | p | q | |
| M – Variance | R_NASDAQ_COMPOSITE | + | yes | Student | 1 | 1 | yes |
| | R_EURO_STOXX_50 | + | yes | Student | 1 | 1 | yes |
| | R_FTSE_100 | + | yes | Student | 1 | 1 | yes |
| | R_SHANGHAI_COMPOSITE | + | No | Student | 1 | 1 | yes |
| | R_TADAWUL | + | yes | Student | 1 | 1 | yes |
| M – Ecart – type | R_NASDAQ_COMPOSITE | + | yes | Student | 1 | 1 | yes |
| | R_EURO_STOXX_50 | + | yes | Student | 1 | 1 | yes |
| | R_FTSE_100 | + | yes | Student | 1 | 1 | yes |
| | R_SHANGHAI_COMPOSITE | + | No[7] | Student | 1 | 1 | yes |
| | R_TADAWUL | + | yes | Student | 1 | 1 | yes |

**Source :** Extraitsd'Eviews 10

With the exception of the R_SHANGHAI_COMPOSITE return series, all other return series are positively related to their corresponding risks. That is, as the risk, measured by conditional variance or standard deviation, increases, the expected return increases, and vice versa.

### 2.1.5. Estimation of EGARCH (p,q) models:

The assumption of linearity in the conditional variance function with respect to the squared errors is not always verified. Nonlinear conditional variance models address this characteristic. The EGARCH model proposes an exponential effect on the conditional variance instead of a quadratic effect. In this study, the EGARCH model is primarily justified by the asymmetry (skewness) of the variance of returns. The EGARCH model is specified by the following formula:

$$\begin{cases} R_{it} = c_i + \varepsilon_{it} \\ \varepsilon_t = \eta_t \sqrt{h_t} \\ h_t = h(\eta_{t-1}, \dots, \eta_{t-q}, h_{t-1}, \dots, h_{t-p}) \\ \ln(\sigma_t^2) = \alpha_0 + \sum_{j=1}^{q} b_i ln(\sigma_{t-j}^2) + \sum_{i=1}^{p} \alpha_i \left|\frac{\varepsilon_{t-i}}{\sigma_{t-i}}\right| + \sum_{k=1}^{r} \alpha_i \frac{\varepsilon_{t-k}}{\sigma_{t-k}} \\ g(\eta_t) = \theta\eta_t + \gamma[|\eta_t| - E|\eta_t|] \end{cases} \quad \dots\dots\dots (7)$$

The results of the estimations are summarized in the following table:

**Table n°09: Estimation result of EGARCH models (p,q,r)**

| | DEE [8]retained | The order | | | Disappearance effect ARCH |
|---|---|---|---|---|---|
| | | p | q | r | |

[6] Simply using the results without reporting the details in the appendices..
[7]Significant at the 10% threshold
[8]Took a GED distribution for the errors following Nelson's initial hypothesis.

| | | | | | |
|---|---|---|---|---|---|
| *R_NASDAQ_COMPOSITE* | GED | 1 | 1 | 1 | yes |
| *R_EURO_STOXX_50* | GED | 1 | 1 | 1 | yes |
| *R_FTSE_100* | GED | 1 | 1 | 1 | yes |
| *R_SHANGHAI_COMPOSITE* | GED | 1 | 1 | 1 | yes |
| *R_TADAWUL* | GED | 1 | 1 | 1 | yes |

**Source :** Extraitsd'Eviews 10

The results show that all five return series can be represented by an EGARCH(1,1,1) model, but the examination of the Jarque-Bera test indicates that the asymmetry effect persists.

### 2.1.6. Estimation of models $TARCH(p,q,r)$ :

Threshold ARCH or GARCH (TARCH) modeling involves incorporating the asymmetry effect into the specification of the conditional variance of errors, where the sign and magnitude of a lagged error shock determine its impact on the conditional variance at time "t." In the modeling of the different return series, the estimation of the TGARCH model is justified by the persistent asymmetry of the errors. The model is specified by the following formula:

$$\begin{cases} R_{it} = c_i + \varepsilon_{it} \\ \varepsilon_t = \eta_t \sqrt{h_t} \\ \sqrt{h_t} = \alpha_0 + \sum_{i=1}^{q} \alpha_i^+ \varepsilon_{t-i}^+ - \sum_{i=1}^{q} \alpha_i^- \varepsilon_{t-i}^- \end{cases} \quad \dots\dots\dots (8)$$

Avec $\varepsilon_t^+ = max\{\varepsilon_t, 0\}$ et $\varepsilon_t^- = min\{\varepsilon_t, 0\}$ under the constraints that $\alpha_0 > 0$ et$\alpha_i^+, \alpha_i^- \geq 0$ to ensure the positivity of the conditional standard deviation.

The results of the estimation are summarized in the following table:

**Table n°10 :** Model estimation results $TGARCH(p,q,r)$

| | DEE retained | The order | | | Signe (r) | Disappearance ARCH effect |
|---|---|---|---|---|---|---|
| | | *p* | *q* | r | | |
| *R_NASDAQ_COMPOSITE* | Student | 1 | 0 | 1 | (+) | yes |
| *R_EURO_STOXX_50* | Student | 0 | 0 | 4 | (+) | yes |
| *R_FTSE_100* | Student | 1 | 0 | 1 | (+) | yes |
| *R_SHANGHAI_COMPOSITE* | Student | 1 | 1 | 2 | (+) et (-) | yes |
| *R_TADAWUL* | Student | 1 | 1 | 0 | ----- | yes |

**Source :**Extraitsd'Eviews 10

With the exception of the TADAWUL index, the positive sign of the "Threshold" term of order r indicates that good news occurring in the respective financial markets increases the volatility of returns.

## 2.2. Criteria and choice of optimal models:

The following tables, from 11 to 15, summarize the results of the different estimated models and the criteria for selecting the optimal model for each of the yield series:

**Table n°11: Choosing the optimal model for (NASDAQ COMPOSITE)**

| Selection criteria | $ARCH(5)$ | $GARCH(1,1)$ | $GARCH(1,1) - I$ | $GARCH(1,1) - I$ | $EGARCH(1,1,1)$ | $TGARCH(1,0,1)$ |
|---|---|---|---|---|---|---|

| | | | | | | |
|---|---|---|---|---|---|---|
| **AIC** | -5,479 | -5,788 | -5,788 | -5,787 | *-5,794* | -5,639 |
| **SHW** | -5,716 | -5,767 | -5,763 | -5,763 | *-5,769* | -5,618 |
| **HQ** | -5,737 | -5,780 | -5,778 | -5,778 | *-5,785* | -5,631 |
| **DW** | 2,097 | 2,098 | 2,077 | 2,083 | *2,103* | 2,099 |
| **Remains ARCH Effect** | No | No | No | No | *No* | No |

**Source :**Extraitsd'Eviews 10

**Table 12: Choice of the optimal model for (R_EURO_STOXX_50)**

| Selection criteria | $ARCH(5)$ | $GARCH$ $(1,1)$ | $GARCH(1,1) -$ | $GARCH(1,1) -$ | $EGARCH$ $(1,1,1)$ | $TGARCH$ $(0,0,4)$ |
|---|---|---|---|---|---|---|
| **AIC** | -5,844 | -5,876 | -5,877 | -5,877 | *-5,899* | -5,844 |
| **SHW** | -5,811 | -5,856 | -5,852 | -5,852 | *-5,874* | -5,815 |
| **HQ** | -5,832 | -5,868 | -5,867 | -5,867 | *-5,890* | -5,833 |
| **DW** | 2,012 | 2,014 | 1,996 | 2,001 | *2,019* | 2,015 |
| **Effet ARCH** | No | No | No | No | No | No |

**Source :**Extraitsd'Eviews 10

**Table 13: Choice of the optimal model for (R_FTSE_100)**

| Selection criteria | $ARCH(5)$ | $GARCH$ $(1,1)$ | $GARCH(1,1) -$ | $GARCH(1,1) -$ | $EGARCH$ $(1,1,1)$ | $TGARCH$ $(1,0,1)$ |
|---|---|---|---|---|---|---|
| **AIC** | -6,229 | -6,243 | -6,245 | -6,245 | *-6,260* | -6,166 |
| **SHW** | -6,196 | -6,222 | -6,220 | -6,220 | *-6,236* | -6,146 |
| **HQ** | -6,217 | -6,235 | -6,235 | -6,235 | *-6,251* | -6,159 |
| **DW** | 2,090 | 2,090 | 2,041 | 2,064 | *2,093* | 2,091 |
| **Effet ARCH** | Non | Non | Non | Non | *Non* | Non |

**Source :**Extraitsd'Eviews 10

**Table 14: Choice of the optimal model for (R SHANGHAI COMPOSITE)**

| Selection criteria | $ARCH(5)$ | $GARCH$ $(1,1)$ | $GARCH(1,1) -$ | $GARCH(1,1) -$ | $EGARCH$ $(1,1,1)$ | $TGARCH$ $(1,1,2)$ |
|---|---|---|---|---|---|---|
| **AIC** | -5,698 | *-5,761* | -5,760 | -5,760 | -5,761 | -5,771 |
| **SHW** | -5,665 | *-5,741* | -5,735 | -5,735 | -5,737 | -5,742 |
| **HQ** | -5,685 | *-5,754* | -5,751 | -5,751 | -5,752 | -5,760 |
| **DW** | 1,986 | *1,986* | 1,986 | 1,986 | 1,987 | 1,987 |
| **Effet ARCH** | No | *No* | No | No | No | No |

**Source :**Extraitsd'Eviews 10

**Table 15: Choice of the optimal model (R_TADAWUL)**

| Selection criteria | $ARCH(1)$ | $GARCH(1,1)$ | $GARCH(1,1)-M(\sigma_t^2)$ | $GARCH(1,1)-M(\sigma_t)$ | $EGARCH(1,1,1)$ | $TGARCH(1,1,0)$ |
|---|---|---|---|---|---|---|
| **AIC** | -5,918 | -5,999 | -5,999 | -5,999 | *-6,008* | ------ |
| **SHW** | -5,902 | -5,978 | -5,975 | -5,974 | *-5,984* | ------ |
| **HQ** | -5,912 | -5,991 | -5,990 | -5,989 | *-5,999* | ------ |
| **DW** | 1,960 | 1,960 | 1,924 | 1,943 | *1,960* | ------ |
| **Effet ARCH** | Non | Non | Non | Non | *Non* | ------ |

**Source :**Extraitsd'Eviews 10

By selecting for each series the model with the minimum values of the information criteria AIC, SC, and QH (underlined in the previous tables), it appears that the EGARCH(1,1,1) model is the most appropriate for modeling the returns of the series R_NASDAQ_COMPOSITE, R_EURO_STOXX_50, R_FTSE_100, and R_TADAWUL, while the GARCH(1,1) model is the most suitable for modeling the returns series R_SHANGHAI_COMPOSITE.

**Table 16 below shows the estimation results of the optimal models:**

| Performance Series | Optimal model | Coefficients | | | | | | DEE |
|---|---|---|---|---|---|---|---|---|
| | | Fonc. Moy. | Variance function | | | | | |
| | | c | $\alpha_0$ | $\alpha_1$ | $\gamma_1$ | $\gamma_2$ | $\beta_1$ | |
| *R_NASDAQ_CO* | EGARCH(1,1,1) | 0,001465 | -0,477 | 0,197 | -0,142 | ---- | 0,961 | GED |
| *R_EURO_STOXX* | EGARCH(1,1,1) | 0,000720 | -0,266 | 0,100 | -0,148 | ---- | 0,978 | GED |
| *R_FTSE_100* | EGARCH(1,1,1) | 0,000440 | -0,249 | 0,073 | -0,166 | ---- | 0,979 | GED |
| *R_SHANGHAI* | GARCH(1,1) | 0,000528 | 0,000 | 0,075 | 0,924 | ---- | ---- | Student |
| *R_TADAWUL* | EGARCH(1,1,1) | 0,001066 | -0,0484 | 0,149 | -0,111 | ---- | 0,957 | GED |

**Table n°16: summary of optimal models**
**Source :**Extraitsd'Eviews 10

### 2.3. Forecasting returns and risks:

The future forecasting of unobservable returns and volatility is crucial for investment decision-making, portfolio management, and risk management. Using the optimal models defined in Table No. 16 to project the returns and volatility until the end of June 2025 (i.e., 96 business days) for the five return series.

The forecasted volatility for the entire period for each return series is aggregated by the average of the predicted daily volatilities, calculated according to the formula:

$$\sigma_T = \sqrt{\frac{1}{T}\sum_{t=1}^{T}\sigma_t^2} \dots\dots\dots (9)$$

And by applying formula (2), the total expected return of the period for each return series, Rt, is the sum of the expected daily returns.

Withe $T = 96$, the number of working days, the forecast results at the end of June 2025 (Appendix No. 14) are summarized in the following table:

**Table No. 17: Expected returns and risks as of June 30, 2025**

| **Performance Series** | **Yield** $R_T$ | **Risk (Ecart-typ** $\sigma_T$**)** |
|---|---|---|
| *R_NASDAQ_COMPOSITE* | 14,06% | 1,62% |
| *R_EURO_STOXX_50* | 6,91% | 1,11% |
| *R_FTSE_100* | 4,22% | 0,72% |
| *R_SHANGHAI_COMPOSITE* | 5,07% | 1,51% |
| *R_TADAWUL* | 10,23% | 1,34% |

**Source :**Extraitsd'Eviews 10

Over the chosen forecast period, the NASDAQ_COMPOSITE stock index has been expected to offer the highest return and risk, at 14.06% and 1.62%, respectively, compared to the other indices. In contrast, the FTSE_100 index offers the lowest expected return of 4.22% and the lowest risk, at 0.72%.]

## 3. **Results and Discussion**

**- Market Efficiency and White Noise Process**

The analysis confirmed that the financial markets under study (represented by the daily returns of the selected indices) exhibit a high level of informational efficiency. This was evidenced by the White Noise process observed in the mean returns function, implying that historical price data cannot reliably predict future returns to generate abnormal gains. This aligns with the **Efficient Market Hypothesis (EMH)**, suggesting that these markets quickly incorporate all available information into asset prices.

**- Volatility Clustering and ARCH Effects**

All return series displayed significant **volatility clustering**, where periods of high volatility are followed by similar periods, and vice versa. The Lagrange Multiplier (LM) tests confirmed the presence of **ARCH effects** (Table No. 05), rejecting the null hypothesis of homoscedasticity. This finding justifies the use of **ARCH-class models** to capture time-varying volatility, as traditional linear models (e.g., ARMA) fail to account for such dynamics.

**- Asymmetry and Leptokurtosis**

- **Asymmetric Returns**: Negative skewness values (Table No. 02) indicated that extreme negative returns occur more frequently than positive ones, particularly in indices like the *Tadawul All Share* (Skewness = -1.847).
- **High Kurtosis**: All series exhibited leptokurtic distributions (Kurtosis > 3), with the *Tadawul All Share* showing the highest kurtosis (21.601). This suggests a higher likelihood of extreme returns than predicted by a normal distribution.

These characteristics necessitated the use of **asymmetric ARCH models** (e.g., EGARCH, TARCH) to account for leverage effects, where negative shocks increase volatility more than positive shocks of the same magnitude.

**- Model Selection and Performance**

The optimal models for each index were selected based on the **Akaike (AIC)**, **Schwarz (SC)**, and **Hannan-Quinn (HQ)** criteria (Tables 11–15):

- **EGARCH(1,1,1)**: Best fit for *NASDAQ Composite*, *Euro Stoxx 50*, *FTSE 100*, and *Tadawul All Share*, capturing asymmetry and volatility persistence.
- **GARCH(1,1)**: Adequate for *Shanghai Composite*, where asymmetry was less pronounced.

Key coefficients from the optimal models (Table No. 16):

- **Persistence of Volatility**: The sum of ARCH and GARCH coefficients (e.g., 0.961 for NASDAQ) confirmed long memory in volatility, indicating shocks dissipate slowly.
- **Leverage Effects**: Negative coefficients for asymmetry terms (e.g., -0.477 for NASDAQ) validated that negative returns increase future volatility more than positive returns.

**- Forecasted Returns and Risks (June 2025)**

The forecasts (Table No. 17) revealed:

- **Highest Return/Risk**: *NASDAQ Composite* (14.06% return, 1.62% risk).
- **Lowest Return/Risk**: *FTSE 100* (4.22% return, 0.72% risk).

These results highlight the trade-off between risk and return, supporting the use of these models for **portfolio construction** and **risk management** in internationally diversified investments.

**- Robustness and Limitations**

- **Non-Gaussian Innovations**: The Generalized Error Distribution (GED) and Student's *t*-distribution improved model fit by accounting for fat tails, though residual asymmetry persisted in some cases.
- **Limitations**: The models assume linear dependencies and may not fully capture structural breaks or regime shifts (e.g., financial crises). Future research could explore hybrid or nonlinear models (e.g., Markov-Switching GARCH).

## 4. Conclusion:

In this research on modeling financial markets, represented by the daily returns of stock indices, the obtained results showed that the corresponding financial markets have reached a

significant level of efficiency, making it challenging to achieve substantial and abnormal gains. These results were reflected in a White Noise process for the mean returns function.
Furthermore, the presence of volatility clustering suggests that achieving additional gains requires examining second-order conditional non-stationarity, which necessitates the use of econometric models better suited to this phenomenon, particularly ARCH-class models. Additionally, the asymmetry observed in all return series made asymmetric ARCH models, especially the EGARCH model, the most appropriate for modeling and forecasting returns.
The high level of kurtosis in the return series compared to the normal distribution required the consideration of alternative probability distributions for innovations in the modeling process. Although the Generalized Error Distribution (GED) reduces the leptokurtic level, it remains persistent, as does the asymmetry. Finally, it is worth noting that the results obtained are comparable to those of other financial indices in Table No. 01.

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