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Optimizing Inventory Management: A Fuzzy Approach to Economic Order Quantity with Constraints

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ABSTRACT
Generally, in deriving the solution of the economic order quantity (EOQ) inventory model, we consider the deterioration rate, holding cost, and ordering cost as constant. But in the case of real-life problems, the above case is not constant but slightly disturbed from its original crisp value. In this paper, a fuzzy inventory model is developed considering deterioration rate, holding cost, and ordering cost as fuzzy variables. Because, in practice, it is not always easy to determine the rate of deterioration precisely. In most cases it is uncertain in nature; therefore, it becomes reasonable to consider the vagueness and uncertainty of the deterioration rate in a fuzzy environment. These variables are represented by the trapezoidal membership function. The function principle is applied to obtain an optimum total fuzzy cost along with optimum order quantity and optimum shortage quantity.

Keywords:
Fuzzy membership function, Fuzzy deterioration rate, Trapezoidal numbers, Function principle, defuzzification.

1. INTRODUCTION
In developing an inventory model, the main goal is to maximize its usefulness. This is closely connected with the relationship between three key characteristics of every system model: complexity, credibility, and uncertainty. Uncertainty is an important aspect of the modeling business, which can be used for gains in the other essential characteristics of models. Fuzzy mathematical programming gained popularity after Zimmermann (1976) introduced the max-min method and applied fuzzy set theory to decision-making. Fuzzy decision making and fuzzy mathematical programming facilitate to develop is an appropriate model for the system in accordance to the current state of information (Rommel Fanger, 1996).

In a fuzzy mathematical programming model, all or some of the parameters can be defined as fuzzy numbers. For the models in which all of the parameters are defined as fuzzy numbers, there are two important things to be ascertained (i) how the relationship between the fuzzy left and the fuzzy right-hand side of the constraints will be handled, and (ii) how the optimal value of the objective function will be found. Recognition of this important role of uncertainty by some researchers become quite explicit in the literature of the 1960s. It is generally agreed that an important point in the evolution of the modern concept of uncertainty was the publication of a seminar paper by Loffi, a. Zadeh (1965 b). In this paper, Zadeh introduced a theory whose objects - fuzzy set. The publication of fuzzy set theory to inventory problem has been proposed by Kacpryzk and Park (1982, 1987). Petrovic (1996) presents a newsboy

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problem assuming that demand and backorder cost are fuzzy numbers. Ishii (1998) introduced the fuzziness of shortage cost explicitly into the classical newsboy problem. Chiant (2002) considered the single-period inventory problem in the presence of uncertainties. Dutta (2005) presented a single inventory problem in an imprecise and uncertain mixed environment. Chang (1999) discussed how to get the economic production model when the quantity of demand is uncertain.

In the real world, minimizing the shortage of items are challenging phenomenon. And also, it is very important to retain the customer despite the shortage of items phenomenon. To address the shortage conditions Chin (2002) introduced the concept of fuzzy inventory models without backorder. Chin (1996) had already introduced the EOQ model with backorder function principles. Li (2006) established the fuzzy inventory model with backorder using trapezoidal fuzzy numbers. Tin the fuzzy inventory model, he total profit per unit time is obtained using fuzzy arithmetic operations, and then defuzzied by the graded mean integration value (GMIV) method (2016). Variation in demand with time and price is one of the major concerns in any inventory system. Several studies report situations where demand varies with time and price separately(2020).

At the initial stage of developing inventory models demand is assumed to be constant and no deterioration of an item in the stock. Inventory models for deteriorating items have been studied by researchers from time to time. Deterioration simply means decay, loss of utility, or marginal value of a commodity that results in the decreasing usefulness of the inventory from the original condition. Inventory models for deteriorating items have been studied by several researchers in recent decades. However, most of the researchers have considered the rate of deterioration as fixed and constant throughout the period of inventory study. In the literature of inventory models for deteriorating items, some of the researchers have expressed deterioration rate as a random variable, and then appropriate statistical distribution(s) have been applied for approximating the rate of deterioration .

In practice, it is not always easy to determine the rate of deterioration precisely . In most cases it is uncertain in nature; therefore, it becomes reasonable to consider the vagueness and uncertainty of the deterioration rate in a fuzzy environment. Due to market strategies, various costs associated with the inventory model cannot be taken as fixed throughout the period. Particularly, inventory holding cost might be affected by some other unseen factors, which may create an unbalance in the budget. Shortage cost is another cost with which the practitioner is concerned. Goodwill and profit maximization are linked to this cost; therefore, it is expected that the shortage cost must be as low as possible. But in practice, due to the competitive situation prevailing in the market, it is highly unlikely to predict the shortage cost.

In this paper fuzzy inventory model is developed considering deterioration rate, holding cost, and ordering cost as fuzzy variables. These variables are represented by the trapezoidal membership function. The function principle is applied to obtain an optimum total fuzzy cost along with optimum order quantity and optimum shortage quantity .

2. METHODOLOGY

2.1 Representation and Mean of Trapezoidal Fuzzy Number

Dubios and Prade (1980, 1982) including other authors have contributed significantly to the body of work on the idea of fuzzy numbers. A fuzzy number in R is defined as a fuzzy subset of R that is convex and normal by Kaufmann and Gupta (1991). As a result, it can be assumed fuzzy number as a generalization of the confidence interval. It is not, however, a random variable. The definition of a random variable comes from the theory of probability, which was developed from the idea of measurement. A fuzzy number is a subjective datum, while a random variable is an objective one. It's not a measure; it's a valuation.

Fuzzy inventory models used for study purposes are trapezoidal fuzzy numbers (TrFN). Suppose $\tilde{A}$ is a trapezoidal fuzzy number as shown in Figure 1. It is described as any fuzzy subset of the real line R, whose membership function $\mu_{\tilde{A}}$ satisfies the following conditions.

(1) $\mu_{\tilde{A}}(x)$ is a continuous mapping from R to the closed interval [0, 1],

(2) $\mu_{\tilde{A}}(x) = 0$, $-\infty < x \le a_1$,

(3) $\mu_{\tilde{A}}(x) = L(x)$ is strictly increasing on $[a_1, a_2]$,

(4) $\mu_{\tilde{A}}(x) = 1$, $a_2 \le x \le a_3$,

(5) $\mu_{\tilde{A}}(x) = R(x)$ is strictly decreasing on $[a_3, a_4]$,

(6) $\mu_{\tilde{A}}(x) = 0$, $a_4 \le x < \infty$,

Where a_1, a_2, a_3, and a_4 are real numbers. Also, this type of trapezoidal fuzzy number be denoted as $\tilde{A} = (a_1, a_2, a_3, a_4)$.
Furthermore, the Graded Mean Integration Representation approach was presented by Chen and Hsieh. This method relies on the integral value of the graded mean h-level of a generalized fuzzy number. It is logical and appropriate to use grade as the significant degree for each fuzzy number point in the generalized point of support set and to talk about the grade of each fuzzy number point in the support set to portray fuzzy numbers. Here, we first use Chen et al.'s technique under w=1 and k-preference by the manager to represent the trapezoidal fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4)$, $P_k(\tilde{A})$,

$$P_k(\tilde{A}) = \frac{k(a_1 + 2a_2) + (1-k)(2a_3 + a_4)}{3} \qquad (1)$$

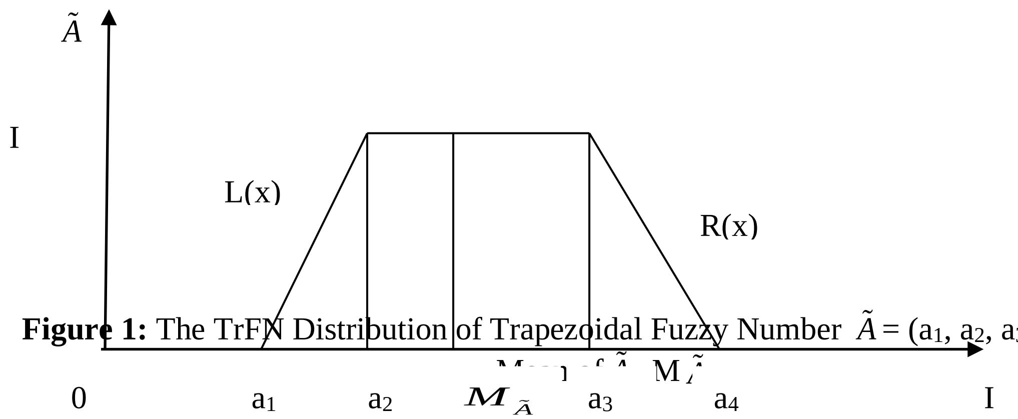

Figure 1: The TrFN Distribution of Trapezoidal Fuzzy Number $\tilde{A} = (a_1, a_2, a_3, a_4)$, and the Mean of $\tilde{A}$, $M_{\tilde{A}}$

Also, the mean of trapezoidal fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4)$, $M_{\tilde{A}}$, by using the method of Chen et al. under w=1 and k=0.5 is defined as shown in Figure 1. However, it is preferred to apply k=0.5, since it is not biased toward either left or right.

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$$M_{\tilde{A}} = \frac{a_1 + 2a_2 + 2a_3 + a_4}{6} \qquad (2)$$

Suppose $\tilde{A}$ = (1, 2, 3, 7) is a trapezoidal fuzzy number, then the representation of $\tilde{A}$ and the mean of $\tilde{A}$ can be calculated by Formula (1) and (2) respectively, as follows.

$$P_k(\tilde{A}) = \frac{5k + 13(1-k)}{3} = \frac{13 - 8k}{3}$$

$$M_{\tilde{A}} = \frac{1+4+6+7}{6}$$

2.2 The Fuzzy Arithmetical Operations under Function Principle

The present study has applied the Function Principle to simplify the calculation. Function Principle in fuzzy theory is used as the computational model to avoid the complications that can be caused by the operations using the Extension Principle. Fuzzy arithmetical operations under the Function Principle have been described as follows.

Suppose $\tilde{A}$ = (a_1, a_2, a_3, a_4) and $\tilde{B}$ = (b_1, b_2, b_3, b_4) are two trapezoidal fuzzy numbers.

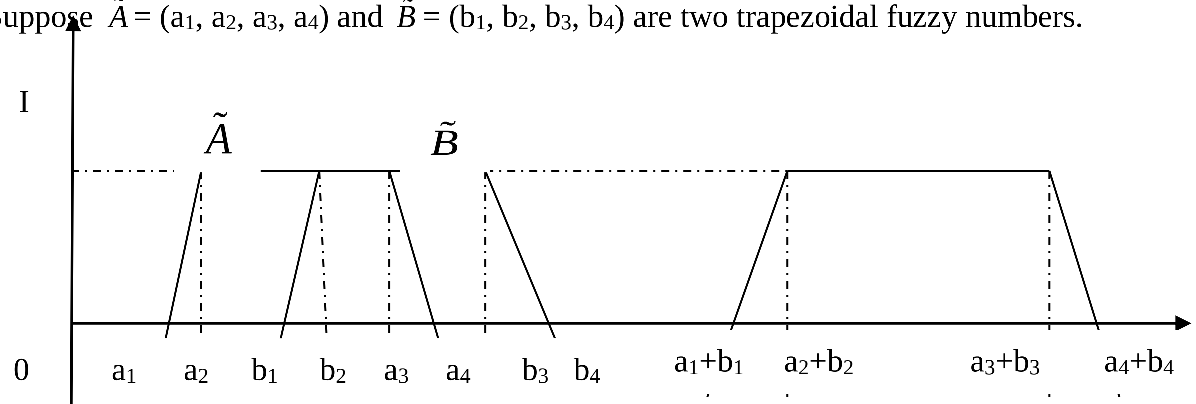
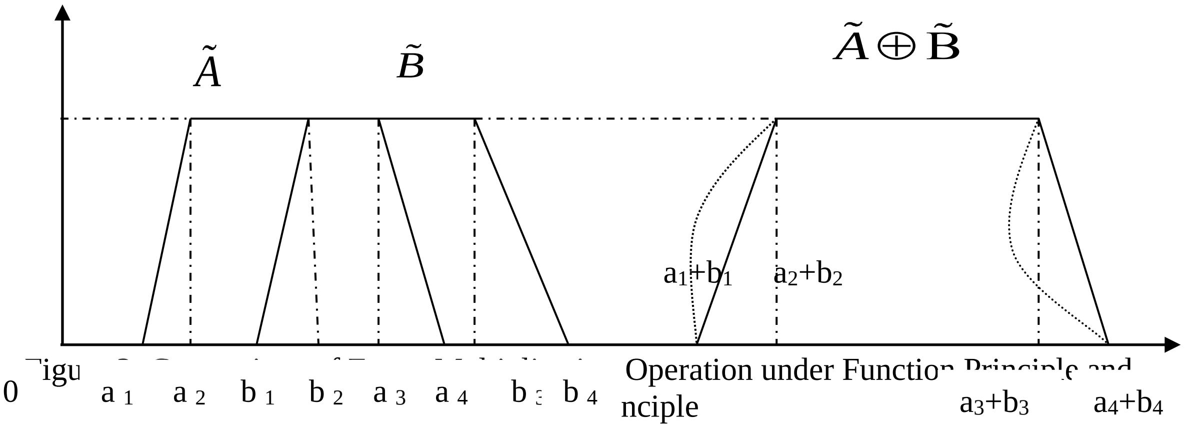


Figure 2: The Fuzzy Addition Operation of Function Principle and Extension Principle



Figu … Operation under Function Principle and … nciple

Then,

(i) The addition of $\tilde{A}$ and $\tilde{B}$ is

$\tilde{A} \oplus \tilde{B}$ = (a_1+b_1, a_2+b_2, a_3+b_3, a_4+b_4),

Where a_1, a_2, a_3, a_4, b_1, b_2, b_3, and b_4 are any real numbers.

(ii) The multiplication of A ~ and $\tilde{B}$ is

$\tilde{A} \otimes \tilde{B}$ = (c_1, c_2, c_3, c_4),

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Where T ={a_1b_1, a_1b_4, a_4b_1, a_4b_4}, T1={ a_2b_2, a_2b_3, a_3b_2, a_3b_3}, c_1 = min T, c_2 = min T_1, c_3 = max T_1, c_4 = max T.
Also, if a_1, a_2, a_3, a_4, b_1, b_2, b_3, and b_4 are all nonzero positive real numbers, then
$\tilde{A} \otimes \tilde{B}$ = (a_1b_1, a_2b_2, a_3b_3, a_4b_4),
Where $\tilde{A} \otimes \tilde{B}$ is a trapezoidal fuzzy number"
(iii) $-\tilde{B}$ = ($-b_4$, $-b_3$, $-b_2$, $-b_1$), then the subtraction of A ~ and B ~ is
$\tilde{A} \Theta \tilde{B}$ = (a_1-b_4, a_2-b_3, a_3-b_2, a_4-b_1),
Where a_1, a_2, a_3, a_4, b_1, b_2, b_3, and b_4 are any real numbers.
(iv)1/ $\tilde{B}$ = $\tilde{B}^{-1}$ = (1/b_4, 1/b_3, 1/b_2, 1/b_1), where b_1, b_2, b_3, and b_4 are all positive real numbers.
If a_1, a_2, a_3, a_4, b_1, b_2, b_3, and b_4 are all nonzero positive real numbers, then the division of $\tilde{A}$ and $\tilde{B}$ is
$\tilde{A} \varnothing \tilde{B}$ = (a_1/b_4, a_2/b_3, a_3/b_2, a_4/b_1).
(v) Let $\alpha \in R$, then

$(i)\alpha \geq 0, \alpha \otimes \tilde{A} = (\alpha a_1, \alpha a_2, \alpha a_3, \alpha a_4)$

$(ii)\alpha < 0, \alpha \otimes \tilde{A} = (\alpha a_4, \alpha a_3, \alpha a_2, \alpha a_1)$

For example, suppose A ~ = (1, 2, 3, 4), $\tilde{B}$ = (1, 3, 4, 6) are two trapezoidal fuzzy numbers, and α=2.5 then
(1) $\tilde{A} \oplus \tilde{B}$ = (2, 5, 7, 10),
(2) $\tilde{A} \otimes \tilde{B}$ = (1, 6, 12, 24),
(3) $\alpha \otimes \tilde{B}$ = (2.5, 7.5, 10, 15),
(4) $\tilde{B} \varnothing \alpha = \tilde{B} \otimes (\alpha\text{-}1)$ = (0.4, 1.2, 1.6, 2.4).

3. MATHEMATICAL MODEL

3.1 Assumptions:

There are the following assumptions made for developing a mathematical model under study:

i) The rate of replenishment is infinite.
ii) The demand rate is fixed and known.
iii) Shortages are allowed.
iv) No repair or replacement of the deteriorating unit.
v) Deterioration of an item is considered only after the items enter in the inventory.

3.2 Notations:

The following notation is used throughout the paper:

i) R = The rate at which demand occurs for each unit of time.
ii) θ = The rate at which the on-hand inventory deteriorates per unit of time.
iii) $\tilde{\theta}$ = Fuzzy deterioration rate of on hand inventory per unit time.
iv) C = Cost of deteriorated unit.
v) c_1= The cost of holding inventory for each unit of time..
vi) $\tilde{c}_1$ = Fuzzy inventory holding cost per unit time.
vii) c_2 = Shortage cost per unit per time.
viii) $\tilde{c}_2$ = Fuzzy shortage cost per unit per time.

ix) c_3= Set-up cost.
x) T = Scheduling time period.

4. INVENTORY MODEL WITH DETERIORATING ITEM

The behavior of the inventory model with the assumptions (as described in the previous section)is depicted in figure 3.

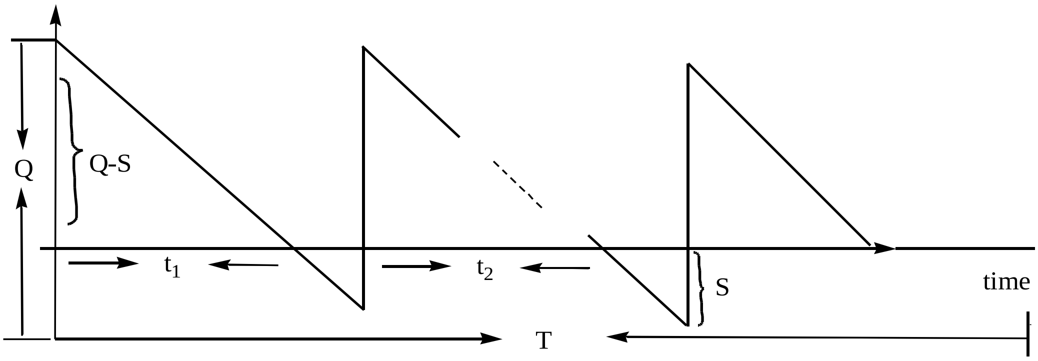

Figure 4: Inventory Model with Shortages

The differential equation that describes the instantaneous states of inventory at a time ' t ' Over the period is given by

$$\frac{dQ(t)}{dt} + \theta Q(t) = -R \qquad 0 \le t \le t_1 \tag{1}$$

$$\Rightarrow D_\theta = \left[\left(Q + \frac{R}{\theta}\right)\left(1 - e^{-\theta t}\right) - Rt_1\right] \tag{2}$$

Solving the differential equation (1) at t=o, Q (t) =Q, we get
The solution of (1) and (2) is given by

$$Q(t) = \frac{-R.}{\theta} + \left(Q + \frac{R}{\theta}\right).e^{-\theta t} \qquad 0 \le t \le t_1 \tag{3}$$

$$Q(t) = R(t_1 - t) \qquad t_1 \le t \le T \tag{4}$$

The total holding cost during (o, t_1) is

$$C_h = c_1 \int_0^{t_1} Q(t)dt$$

$$\Rightarrow C_h = \frac{c_1}{\theta}\left[\left(Q + \frac{R}{\theta}\right)\left(1 - e^{-\theta t}\right) - Rt_1\right] \tag{5}$$

Total deteriorating cost during (o, t_1) is

$$\Rightarrow D_\theta = \left[\left(Q + \frac{R}{\theta}\right)\left(1 - e^{-\theta t}\right) - Rt_1\right] \tag{6}$$

$$C_h = c_1 \int_0^{t_1} Q(t)dt$$

$$\Rightarrow C_h = \frac{c_1}{\theta}\left[\left(Q + \frac{R}{\theta}\right)\left(1 - e^{-\theta t}\right) - Rt_1\right] \tag{7}$$

Total shortage cost,

$$C_s = c_2 \int_{t_1}^{T} R(t_1 - t)dt$$

$$C_s = -\frac{c_2 R}{2}\left(t_1 - T\right)^2$$

Total Inventory Cost (TIC) = Total Holding cost (THC) + Total Shortage cost (TSC) + Total Deterioration Cost (TDC)

$$TIC = \frac{c_1}{\theta}\left[\left(Q + \frac{R}{\theta}\right)\left(1 - e^{-\theta t}\right) - Rt_1\right] - \frac{c_2 R}{2}\left(t_1 - T\right)^2 + \left[\left(Q + \frac{R}{\theta}\right)\left(1 - e^{-\theta t}\right) - Rt_1\right] \qquad (8)$$

Solving equation (8) with respect to t_1 and equating to zero, we get,

$$\frac{\partial TC}{\partial t_1} = 0$$

$$t_1 = \frac{1 - \frac{C_1 R}{\theta} + C_2 T}{C_2}$$

Solving equation (8) with respect to T and equation to zero, we get,

$$\frac{\partial TC}{\partial T} = 0$$

$T = t_1$

5. FUZZY MATHEMATICAL MODEL

In the crisp model (as described in the previous section), it was assumed that all the parameters were fixed or could be predicted with certainty, but in practice due to unseen factors, the parameters of the model could not be assumed to be constant throughout the period .

Usually rate of deterioration is vague in nature; therefore, it is reasonable to consider it into fuzzy environment. Thus, instead of considering the rate of deterioration as constant, the EOQ model is developed with the assumption that the deteriorating rate is a fuzzy number. Similarly, inventory holding cost and shortage cost cannot be considered fixed throughout the horizon. Therefore, the fuzzy number represents both the cost .

In this model, the deteriorating rate, shortage cost, and inventory holding cost are represented by trapezoidal fuzzy numbers. By using the function principle, a fuzzy total average cost is obtained.

Suppose that, in the equation 14 and 17 $\tilde{\theta}, \tilde{c}_1, and\ \tilde{c}_2$ are fuzzy numbers with the trapezoid membership function defined as:

$$\left.\begin{aligned}\tilde{\theta} &= (\theta_1, \theta_2, \theta_3, \theta_4) \\ \tilde{c}_1 &= (c_{11}, c_{12}, c_{13}, c_{14}) \\ \tilde{c}_2 &= (c_{21}, c_{22}, c_{23}, c_{24})\end{aligned}\right\} \tag{9}$$

By function principle, the membership function of fuzzy total average cost is given by $\tilde{K} = (k_1, k_2, k_3, k_4)$, where,

$$k_i = \frac{R}{2T}\left[\left(c_{2i} * (T - t_1^2) - c_{1i} * t_1^2\right)\left(\theta_i * t_1^2 (C - c_{1i} * t_1)\right)\right] + c_3 / T$$

Where i =1, 2, 3, 4 (10)

Similarly, an optimum value t_1 is to be obtained from (17) as follows:

$$(c_{1i}^2 + c_{1i} * c_{2i}) t_1^2 + \left(\theta_i * t_1^2\right)\left[C(2c_{1i} + c_{2i}) + c_{1i} * t_1\left(c_{1i} - c_{2i}/3\right)\right] - c_{2i} * \frac{2c_3}{R} = 0 \tag{11}$$

By using the median rule, (10), (11) and (12) are rewritten as:

$$T_m = \frac{\frac{1}{4}\left[Rt_1\left(\sum_{i=1}^{4} c_{1i} - \sum_{i=1}^{4} c_{2i}\right) + \frac{c_3}{T} + CRt_1\left(\sum_{i=1}^{4}\theta_i\right) - \left(\sum_{i=1}^{4} c_{1i} * \theta_i\right)\left(Rt_1^2/2\right)\right]}{R\left(\sum_{i=1}^{4} c_{2i}\right)} \quad \ldots (12)$$

$$K_m = \frac{1}{4}\left[\left(\sum_{i=1}^{4} c_{2i}\right)\frac{R}{2T}(T - t_1^2) + \frac{c_3}{T} - \sum_{i=1}^{4} c_{1i}\left(\frac{Rt_1^2}{2T}\right) + \sum_{i=1}^{4}\theta_i * \left(\frac{R}{2T}(Ct_1^2 - \sum_{i=1}^{4} c_{1i} * t_1^3)\right)\right] \quad \ldots (13)$$

$$\left(\sum_{i=1}^{4}\theta_i * c_{1i}\right)\left\{\left(\sum_{i=1}^{4} c_{1i}\right) - \left(\sum_{i=1}^{4} c_{2i}/3\right)\right\} t_1^3 + \left[\left(\sum_{i=1}^{4} c_{1i}^2 + \sum_{i=1}^{4} c_{1i} * c_{2i}\right) + \left(C\sum_{i=1}^{4}\theta_i\right)\left(2\left(\sum_{i=1}^{4} c_{1i}\right) + \sum_{i=1}^{4} c_{2i}\right)\right] - \frac{2c_3}{R}\left(\sum_{i=1}^{4} c_{2i}\right) = 0 \quad \ldots (14)$$

Now equation 14 can be solved for the optimum value of t_1, by the Newton-Rapson method. Using this value of t_1 in the equation 13, the optimum value for T can be obtained. The values of T and t_1 will be used to calculate the optimum fuzzy total cost from (12).

6. CONCLUSION

The present study discusses the importance of inventory management in a fuzzy environment. Previous researchers applied the EOQ model taking into account the ordering cost, holding cost, and deterioration rate as more or less fixed quantities. In this research, an EOQ inventory model with a fuzzy system was developed. For parameters such as deterioration rate, holding cost, and ordering cost, the membership functions were established. This study developed a fuzzy inventory model for degrading items, taking into account a range of costs and deterioration rates as fuzzy parameters to accommodate real-world scenarios. Additionally, a sensitivity analysis of the model's various parameters has been done. With modifications to the model's various parameters, Table 1 displays the highly sensitive parameters and Table 2 displays the less sensitive or un-sensitive parameters of the decision variables q and Z .

7. REFERENCES:

1. Chang, S. C., (1999). Fuzzy Production Inventory for Fuzzy Product Quantity with Triangular Fuzzy Number, Fuzzy Sets and Systems, **107**, pp.37-57.
2. Chung, K. H., Liao, C. J., & Lin, F. Y. (2015). Fuzzy economic order quantity inventory model with partial backorders. Fuzzy Economic Review, 20(3), 123-144.
3. Chen, S. H. and C. H. Hsieh, (1999). Optimization of Fuzzy Backorder Inventory Models, *1999 IEEE International SMC'99 Conference (Tokyo) Proceedings*, **1**, pp.296-301.
4. Chen, S. H. and C. H. Hsieh, (1999). Optimization of Fuzzy Simple Inventory Models, *1999*
5. Chen, S. H. and C. H. Hsieh, (2000). Representation, Ranking, Distance, and Similarity of L-R Type Fuzzy Number and Application, *Australian Journal of Intelligent InformationProcessing Systems*, **6**, pp.217-229.
6. Chen, S. H., (1985). Fuzzy Linear Combination of Fuzzy Linear Functions under Extension
7. Chen, S. H., (1985). Operations on Fuzzy Numbers with Function Principle, *Tamkang Journal of Management Sciences*, **6**, pp.13-26.
8. Chen, S. H., C. C. Wang, and A. Ramer, (1996). Backorder Fuzzy Inventory Model under
9. Dubois, D. and H. Prade, (1980). Fuzzy Sets and Systems: Theory and Applications, Academic Press, New York, pp.71-73.
10. Dubois, D. and H. Prade, (1982). A Class of Fuzzy Measures Based on Triangular Norms-A *Optimization of Fuzzy Inventory Models under Fuzzy Demand and Fuzzy Lead Time* general Framework for The Combination of Uncertain Information, *International Journal Gen. Systems*, **8**, pp.43-61.
11. Function Principle, *Information Sciences - an International Journal*, **95**, pp.71-79.
12. Hsieh, C. H., (2002). Optimization of Fuzzy Production Inventory Models, *Information Sciences - An International Journal*, **146**, pp.29-40.
13. Hsieh, C. H., 2003. Optimization of Fuzzy Inventory Models Under k-preference, *Tamsui Oxford Journal of Mathematical Sciences*, **19**, pp.79-97.
14. *IEEE International Fuzzy System Conference (Seoul, Korea) Proceedings*, **1**, pp.240-244.
15. J. Kacpryzk, P. Stanie Wski (1982). Long term inventory policy-making through fuzzy decision – making models. *Fuzzy sets and systems*, Vol. 8, pp.117 – 132.
16. Jimenez, M., (1996). Ranking fuzzy numbers through the comparison of its expected intervals. *International Journal of Uncertainty*, Fuzziness and Knowledge-Based Systems, 4(4), 379-388.

17. Jimenez, M., Arenas, M., Bilbao, A., Rodriguez, M.V., (2007). Linear programming with fuzzy parameters : An interactive method resolution. *European Journal of Operational Research*, Vol.177.
18. Jimenez, M., Rodriguez, M.V., Arenas, M., Bilbao, A., (2000). Solving a possibilistic linear program through compromise programming. *Mathware and Soft Computing*, 7, 175-184.
19. K.S. Park, (1987). Fuzzy set theoretic interpretation of economic order quantity. IEEE Transactions on systems. *Man and cybernetics*, pp. 1082.
20. Kacprzyk, J. and P. Staniewski, (1982). Long-Term Inventory Policy-Making through Fuzzy Decision-Making Models, *Fuzzy Sets and Systems*, **8**, pp.117-132.
21. Kaufmann, A. and M. M. Gupta, (1991). Introduction to Fuzzy Arithmetic Theory and Applications, Int. Thomson Computer Press, USA.
22. Narasimhan, S. L., D. W. Mcleavey, and P. J. Billington, (1995). Production Planning and Inventory Control, New Jersey.

23. Park, K. S., (1987). Fuzzy Set Theoretic Interpretation of Economic Order Quantity, *IEEE Transactions on Systems, Man, and Cybernetics SMC-17*, pp.1082-1084.
24. Principle and Second Function Principle, *Tamsui Oxford Journal of Management Sciences*,**1**, pp.11-31.
25. Roy, T. K., Ray, J., & Jana, D. (2020). Fuzzy inventory model for deteriorating items with time and price-dependent demand with partial backlogging under crisp and fuzzy environment. Annals of Operations Research, 1-29
26. S. K. Indrajitsingha, P. N. Samanta, and U. K. Misra, “Fuzzy inventory model with shortages under fully backlogged using signed distance method,” *International Journal for Research in Applied Science & Engineering Technology*, vol. 4, pp. 197–203, 2016.
27. Wang, C. C., (1997). Quasi-Fuzzy Computing for Decision Methods. Ph.D Thesis, UNSW, Sydney, Australia.
28. Wang, P. P., (1997). Advances in Machine Intelligence & Soft-Computing IV, USA, pp.242-252.
29. Ullah, S., Khan, S. A., & Shah, A. G. (2018). Fuzzy economic order quantity models with combined constraints of budget and storage space. International Journal of Fuzzy Systems, 27(3), 609-621.
30. Yao, J. S., Chiang, M. H., & Wu, C. W. (2012). Fuzzy economic order quantity model with fuzzy demand and fuzzy cost under the possibility measure. Mathematical Problems in Engineering, 2012.

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