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Two Asset Portfolio Optimizations With GARCH Models

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Abstract
The asymmetric information to investors creates, unlike responses to the news. The study is based on GARCH models, which are usually used to analyse the volatilities
and covolatilities in different markets. This paper will apply the concept of an asymmetric dynamic conditional correlation model to calculate conditional fluctuations using the ADCC model, correlations, optimal weights, and hedge ratios, as proposed by Kroner and Neg model, for real estate, crude oil, gold, and the Indian equity index. The study will also help us to optimise the portfolio without compromising the return, which is limited to two asset portfolios.

Keywords: Financial Market, Volatility, Hedge Ratio, Optimum Weight Ratio, portfolio, Asset Class, GARCH models

1. Introduction
Volatility in stock and commodity markets often reacts asymmetrically to positive and negative news, impacting profit levels significantly. This article examines the relationships among returns, volatility correlations, hedge ratios, and optimal portfolio weights derived from the variance and covariance of various asset classes. The study emphasizes two key parameters in financial transactions to mitigate risk: the hedge ratio, which accounts for market asymmetry, and the correlation of volatilities among commonly traded commodities and alternative investments, such as real estate. We explore non-traditional models for estimating volatilities and hedge ratios.
Given that correlations fluctuate over time, we argue that a short-term analysis is essential to understand how return volatility may increase in a bear market, particularly in relation to other market volatilities. Accurate estimation of the covariance matrix and correlation structure is crucial for portfolio and risk managers. This study focuses on the short-term returns observed in 2020, during which the global spread of COVID-19 affected all asset classes. We aim to determine if this financial uncertainty resulted in heightened correlations among capital market assets and whether an asymmetric DCC-GARCH model offers a better fit for understanding these relationships and the contagion spread.

2. Literature Review
Multivariate GARCH models are commonly employed to analyze volatilities and covolatility across markets, with Kearney and Patton (2000) noting their effectiveness in measuring how news influences market fluctuations. Cappiello, Engle, and Sheppard (2006) introduced an asymmetric DCC model to assess how changes in asset correlation exhibit asymmetric responses to negative returns. Investigating correlations among well-known asset classes—

such as the NSE index, gold, crude oil, and real estate—can provide valuable insights for investors seeking to hedge their portfolio exposures.

3. Gap

A review of existing literature on optimal portfolios using GARCH models reveals that most studies focus on different indexes rather than various asset classes. This research will analyze four distinct asset classes: the Nifty Fifty index, gold, crude oil, and real estate. All these asset classes belong to different segments within the financial market. The analysis will utilize data from the Indian financial market.

While management decisions regarding capital structure and budgeting differ from portfolio analysis and financial planning, our two-asset optimization model will employ capital budgeting techniques to identify the most profitable long-term investments that enhance business operations.

4. Objectives

To explore the application of GARCH models in financial markets.

To implement the ADCC model for calculating conditional volatilities.

To compute and assess risk-minimizing hedge ratios for a two-asset portfolio.

To determine optimal portfolio weights that minimize risk for any two-asset combination.

Scope: This study could be expanded into a comparative analysis across two different countries to evaluate how investor preferences vary in each context.

5. Methodology

This section outlines the methodology for modeling time-varying volatility and correlation of variables before analyzing optimal portfolio weights. We will detail how to compute optimal hedge ratios and assess their effectiveness post-portfolio construction.

The GARCH updating formula utilizes a weighted average of unconditional variance, squared residuals from the first observation, and starting variance to estimate subsequent variances, ultimately creating a comprehensive time series of variance forecasts.

We will apply the asymmetric dynamic conditional correlation (ADCC) model introduced by Cappiello et al. (2006) to estimate conditional volatility, correlations, optimal weights, and hedge ratios for real estate, crude oil, gold, and the Indian equity index. Recent literature indicates that the ADCC model is particularly suited for estimating conditional correlation and covariance among diverse asset time series due to its ability to accommodate dynamic correlation and the asymmetric behavior of stock markets.

5.1. Asymmetric Dynamic Conditional Correlation (ADCC) Model

The ADCC model, initially proposed by Cappiello et al. (2006), involves a two-step estimation process. The first step estimates conditional variances using random error terms derived from a conditional mean model, applying a VAR model to account for autocorrelations. The second step employs a VARMA-GARCH (1, 1) approach to model conditional variances and covariances. This method effectively captures volatility spillovers, unlike a standard GARCH model, allowing for insights into how shocks in one variable affect others.

In the first step, the mean equation is modeled as a VAR with one lag, where ritr_{it}rit is the vector of daily returns for oil, gold, and real estate. The model calculates these returns

based on closing prices. The second step involves estimating conditional correlations based on standardized residuals from the first step, resulting in a conditional covariance matrix.
The ADCC model will be estimated using Quasi Maximum Likelihood Estimation (QMLE) with a BFGS algorithm, computing T statistics through a robust covariance matrix.

5.2 The time-varying conditional correlation coefficient

$$Q_{ijt} = \frac{E[\epsilon_{it}\epsilon_{jt}]}{\sqrt{E[\epsilon_{it}^2]E[\epsilon_{jt}^2]}}$$

Where:
Qijt represents the conditional correlation between assets iii and jjj at time ttt.
ϵit and ϵjt are the standardized residuals from the first step of the ADCC estimation, indicating the deviations of asset returns from their conditional means.
An asymmetric DCC model will be estimated using a Quasi Maximum Likelihood Estimation (QMLE) with the Broyden-FletcherGold farb-Shanno (BFGS) algorithm and the T statistics being computed by a robust estimate of the covariance matrix.

5.3. Optimal Portfolio Weights
To construct an optimal portfolio that minimizes risk while maintaining expected returns, we define weights for a two-asset portfolio. The optimal weights will be calculated according to established financial theories.

$$w_{ij,t} = \frac{w_j}{\sum_j w_j}$$

5.4. Optimal Hedge Ratios
To minimize risk, we will adopt methods from Kroner and Sultan (1993) for determining risk-minimizing hedge ratios in a two-asset portfolio.

$$\tau_{ij,t} = \frac{h_{i,t}}{h_{j,t}} \cdot \mathrm{Cov}(R_i, R_j)$$

Where:
τij,t is the optimal hedge ratio between asset iii and asset jjj.
hi,t is the conditional variance of asset iii.
hj,t is the conditional variance of asset jjj.
Cov(Ri,Rj) is the conditional covariance between the returns of asset iii and asset jjj.

5.5. Measuring Performance of a Hedged Portfolio
We will apply the hedge effectiveness index proposed by Ederington (1979) to evaluate the performance of the hedged portfolio. This index compares the variance of a hedged portfolio (involving both long positions in underlying assets and short positions in futures contracts) to that of an unhedged portfolio.

Hedge Effectiveness

- Hedge Ratio: The proportion of the portfolio that is hedged. It can be assessed by comparing the changes in the value of the hedged portfolio against the unhedged portfolio.
- Hedging Effectiveness Ratio:
- Effectiveness=ΔVhedged/ΔVunhedged
- A value closer to 1 indicates a more effective hedge.

6. Data

The data used in this paper includes daily closing values of: MCX oil future and gold future and National Stock exchange of India's index Nifty 50 from nseindia.com and mcx.com for nearby futures' contract of gold (GOLD) and nearby futures' contract of Brent crude oil (OIL) and nifty fifty nearby futures. In addition to this Real Estate asset class data is also collected and used for analysis. The prices of all commodities are expressed in INR. The sample period is from 1st April 2016 to March 31, 2023. The choice of the period is meant to capture the effects of the European debt crisis and pre and post-COVID-19 global pandemic crisis. We make use of daily closing price data to capture the intensity and speed of the dynamic transmission between commodity and stock markets' returns. In total, our analysis includes 434 observations. Continuously compounded daily returns are obtained from stock indices and futures price of commodities as:

$Pt=100*Ln(P_t/P_{t-1})$

7.Estimation and discussion of results

The figure shows the squared returns of each variable, which represent their unconditional volatilities or variances. While all variables depict high volatilities during main economic and financial crises, such as the Jan 2008–Oct 2009 global financial crisis, the Jan 2010–Oct 2011 European debt crisis, and the current COVID-19 global pandemic crisis, crude OIL has displayed high volatilities during COVID-19 global pandemic crisis. The price of Brent crude oil dropped spectacularly to a Low of 1.52K INR in April 2020 and reached a high of 9.12k INR in June 2022 due to the low demand for oil caused by the lockdown. The nifty fifty index has dropped spectacularly to a low of INR 77.35 K and reached a high of INR 18.887K in Dec 2022.

Figure 1: Nifty 50 Index

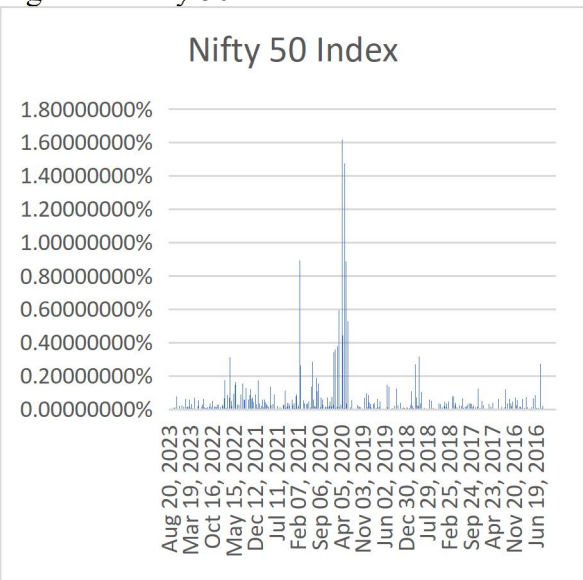

Figure2: Gold

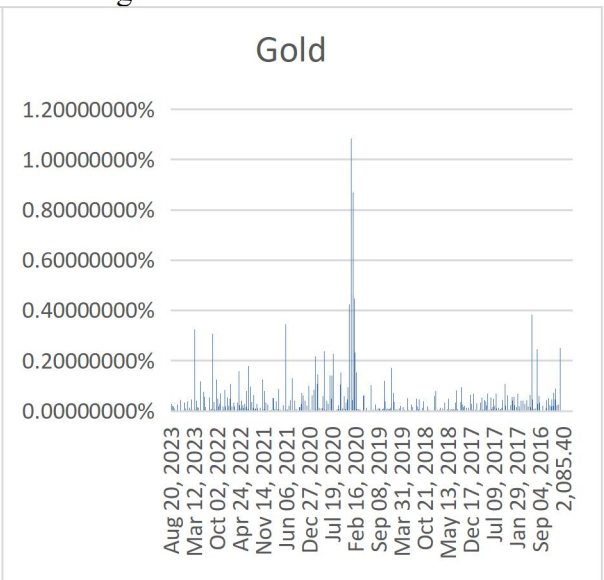

Figure 3: Crud e Oil

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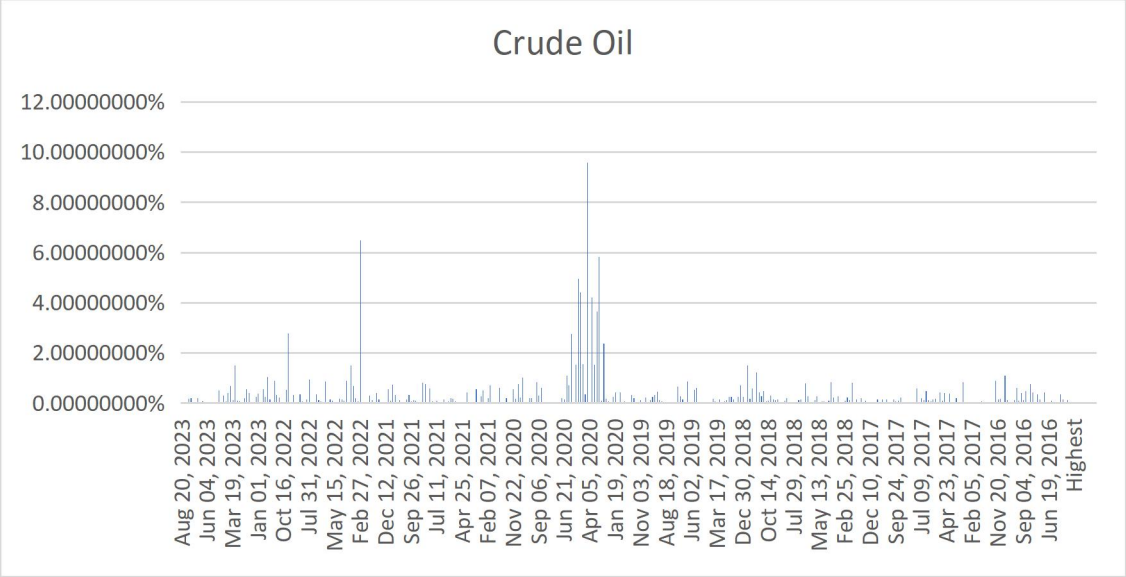

One more indicator is also used to analyze the correlation of different asset classes. That is Real estate. This asset class has different features as compared to NSE and commodity market products. The correlation of returns and unconditional Volatility of real estate with NSE is also considered for better analysis and results of the study.

Moreover, Table 1 shows the correlation between the returns of different variables and Table 2 is a correlation between unconditional volatilities.

The table indicates the extent of the volatility connection among stocks and commodities. The results show high correlations between stocks' volatilities, especially the crude oil and NSE Index' volatilities, and data are collected from April 2016 to August 2023. For example, the correlation between the market of Crude Oil and NSE is 0.112359, the correlation between the NSE and Gold is -0.08235, and the correlation between the NSE and real estate 0.030843.

Table 1:

Correlation of Returns

	Nifty	Crude oil	Gold	Real estate
Nifty	1			
Crude oil	0.112359	1		
Gold	-0.08235	-0.13089	1	
Real estate	0.030843	-0.04094	-0.21875	1

There is a low correlation between stocks and commodities. For example, the volatility correlation between NSE and GOLD is 0.3207, NSE and crude oil is -0.429105 and NSE and real estate is 0.031912. The volatility correlation between Nifty and crude Oil is relatively higher compared to other assets. While the results of the correlation of unconditional volatilities are

Table 2

Volatility correlation

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	Nifty	Crude oil	Gold	Real Estate
Nifty	1			
Crude oil	0.429105	1		
Gold	0.3207	0.273682	1	
Real estate	0.031912	-0.0413	-0.21598	1

Table 3

Hedge ratio

	Nifty	Crude oil	Gold	Real estate
	0.05%	0.32%	0.04%	10.00%
Nifty		-0.07282	.002963	3.242233274
Crude oil	-0.01152		0.00249	2.44254152
Gold	0.003934	-0.02092		0.45713624
Real estate	0.016367	-0.07797	-0.00174	

Indicates the hedge ratios for different pairs of asset class. The highest hedge ratio is nifty and real estate that is 3.242233274. lowest hedge ratio is in between gold and real estate that .00174.

Table 4:

Optimum portfolio weight

Two asset Portfolio	Optimum weight	Optimum weight ratio as per Kroner and Neg
Nifty and crude oil	11%	0.11
Nifty and gold	57%	0.57
Nifty and real estate	43%	0.43
gold and crude oil	17%	0.17
crude oil and real estate	1%	0.01
gold and real estate	51%	0.51

7.1 **Final discussion on result**

Fig 1 analysis shows the squared returns of each variable, which represent their unconditional volatilities or variances. The periods, that show shocks in the history of the last 5 years for all three pillars of the market are, such as the Gold in 2019 from a high of $1550 in September 2019 and further peak in A, 2020 at $2051 to a low $1671in March 2021 and similarly next peak was in March 2022 at $2082, and crashed to $1681 in October 2022, Crude Oil in 2018 from high Rs13157 in September 2018 to low Rs 7735 in December. 2018, similarly in next peak was in April 2019 at Rs. 10515 and crashed to 1959 in April 2020 due to the low demand for oil caused by lockdown measures adopted by many countries to fight the pandemic. Similarly, for NSE equity Index of India has shown a peak of 12168 in December 2019 and dropped to 8579 in April 2020 due to fear of the pandemic. However, it recovered very soon and showed a new peak of Rs.18875 in December 2022.

Table 1 indicates the extent of volatility connection among stocks Market indexes and commodities. The results show high correlations between stocks between nifty index and

crude Oil. In addition to this, gold has the highest correlation with real estate which is .021875.
Table 2 indicates the volatility variance of different asset classes.
Table 3 Indicates the hedge ratios for different pairs of asset classes. The highest Table 3 The best plausible strategy to meet the risk is a hedge ratio that minimizes the losses.
Table 4 indicates the optimum portfolio weights for two asset portfolios. As per the table results maximum weight is for the Nifty and Gold portfolio and Minimum optimum ratio is for Crude Oil and Real Estate.
There is a low correlation between stocks and commodities. For example, the return correlation between Nifty and GOLD is -0.0823. The volatility correlation between Nifty and commodities Crude Oil is relatively higher compared to other assets. The hedge ratio is comparatively low for gold and high for real estate with other asset classes.

7.2 Detailed Explanation: Companies can make their long-term investment better and improve the operations of the business by applying capital budgeting techniques on the five pairs of assets along with their optimum weight ratios.
As per my study gold and Nifty portfolio will give handsome returns with the lowest risk. Returns from nifty investments are increased and risk is decreased by diversification of investment through gold. Table 5 is an example of 5-year data of nifty and gold.
Table 5:

Particulars	Value
Gold price as of 1 April 2018	3148
Nifty as of 1 April 2018	10151
Gold price as of 31 March 2023	5427
Nifty as of 31 March 2023	17381
Gold CAGR for 5 years	0.115077
Nifty CAGR for 5 years	0.113559
Optimum portfolio ratio	0.57
Return of portfolio	0.114212
Risk of Gold	0.0194
Risk of Nifty	0.0224
Risk of portfolio	0.0146

7.3 Application for Management:
The study has application in treasury management of the company where the raw material or finished goods price volatility is high and governed by risk factors of financial, commodity, and forex market. The objective of every management is to invest in the lower-risk area without compromising the return.
The study suggests how to select the best asset class to hedge and business risk and meet the risk associated with investing in the area where volatility correlation is negative and which reduces the risk associated with a portfolio without reducing the return.
This study suggests that the best asset classes for the two asset portfolios are Nifty and Gold because their correlation is negative, optimum portfolio weight is also highest and gold is the perfect asset for hedging.

8. References
1. H. Markowitz, "Portfolio selection," Journal of Finance, vol. 7, pp. 77-91, 1952.

2. H. Konno, "Piecewise linear risk function and portfolio optimization." Journal of the Operations Research Society of Japan, vol. 33, pp. 139-156, 1990.
3. W. Hallerbach and J. Spronk, "A multidimensional framework for portfolio management." in Essays in decision making, Berlin Heidelberg., Springer, 1997, pp. 275-293.
4. E. Ballestero, "Approximating the optimum portfolio for an investor with particular preferences," Journal of the Operational Research Society, vol. 49, pp. 998-1000, 1998.
5. R. Engle and G. Lee, "A Permanent and Transitory Component Model of Stock Return Volatility," 1999.
6. R. Engle, "GARCH 101: The use of ARCH/GARCH models in applied econometrics," Journal of Economic Perspectives, pp. 157-168, 2001.
7. Shiqing Ling and Michael McAleer, “Asymptotic Theory for a Vector ARMA-GARCH Model,” Cambridge University Press, 2003.
8. Cappiello, L., Engle, R.F., and Sheppard, K. (2006), "Asymmetric Dynamics in the Correlations of Global Equity and Bond Returns," Journal of Financial Econometrics, 4, 537-572.
9. W. Chkili, "Dynamic correlations and hedging effectiveness between gold and stock markets: Evidence for BRICS countries," Research in International Business and Finance, Volume 38, 2016.
10. Gaies, M., Chkili, W, "Dynamic correlation and hedging strategy between Bitcoin prices and the stock market during the Russo-Ukrainian war," Eurasian Economic Review, Volume 13, Issue 2, June 2023.