

# Numerical Simulation of Unsteady Hydromagnetic Flow in Channel Flows with Variable Conductivity

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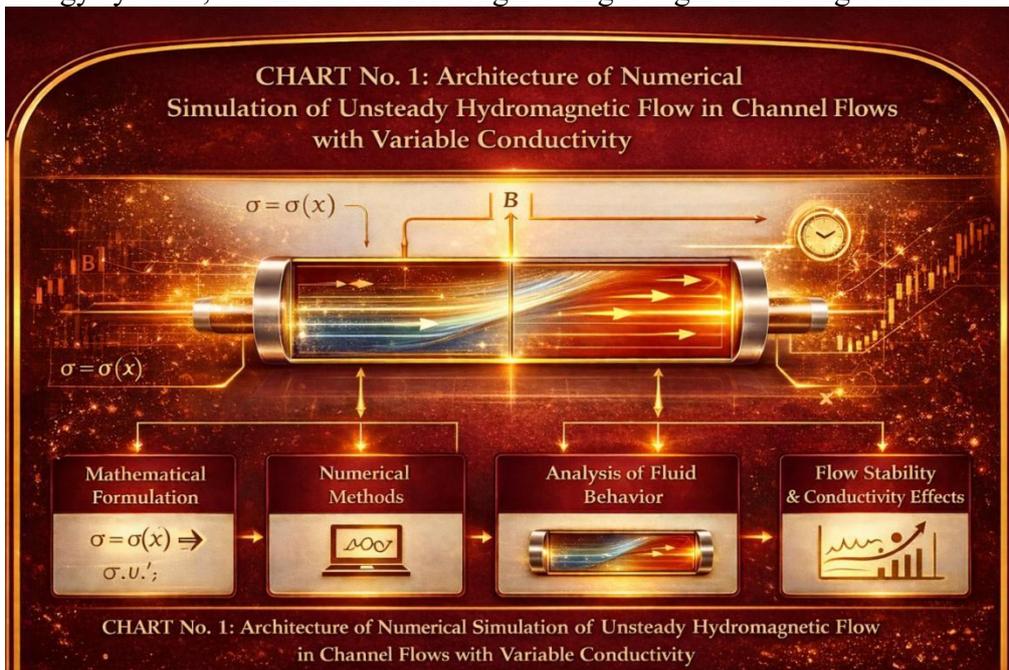
## Abstract

The study of magnetohydrodynamic (MHD) flows has gained considerable importance due to its wide range of applications in engineering, industrial processing, and geophysical fluid dynamics. The present research investigates the numerical simulation of unsteady hydromagnetic flow in channel flows with variable electrical conductivity. The primary objective of this work is to analyze how variations in electrical conductivity influence the velocity distribution, magnetic field interaction, and overall flow characteristics within a channel subjected to an external magnetic field. Understanding these effects is essential in applications such as cooling systems of nuclear reactors, MHD power generators, metallurgical processes, and plasma flow control. A mathematical model describing the unsteady incompressible flow of an electrically conducting fluid between two parallel plates is formulated using the governing equations of momentum and continuity. The presence of a transverse magnetic field introduces Lorentz forces, which significantly affect the velocity profile and fluid motion. The model also incorporates spatial variations in electrical conductivity to represent realistic physical conditions. To solve the resulting nonlinear partial differential equations, suitable numerical techniques are applied, enabling the computation of velocity distributions and flow behavior under different parameter conditions. The numerical results reveal that the magnetic field strength and conductivity variation play a crucial role in controlling the flow structure. An increase in the magnetic parameter tends to reduce the fluid velocity due to the resistive Lorentz force acting against the flow direction. Additionally, variations in electrical conductivity alter the distribution of magnetic forces within the channel, leading to noticeable changes in flow stability and velocity gradients. The unsteady nature of the flow further highlights the time-dependent development of the velocity field before reaching a steady state. Overall, the study provides valuable insights into the complex interaction between magnetic fields and conductive fluids in channel flows. The findings contribute to a better understanding of hydromagnetic transport phenomena and may assist in optimizing engineering systems where magnetic field control of fluid motion is required.

## Introduction

Magnetohydrodynamics (MHD) deals with the study of the motion of electrically conducting fluids under the influence of magnetic fields. Such fluids include liquid metals, electrolytes, ionized gases, and plasmas, which are commonly encountered in many scientific and engineering applications. The interaction between magnetic fields and conductive fluids produces electromagnetic forces that significantly influence the fluid flow characteristics. Because of its wide practical importance, magnetohydrodynamic flow has attracted considerable attention from researchers in fields such as energy generation, nuclear engineering, metallurgical processes, and cooling technologies. Understanding the behavior of hydromagnetic flows is therefore essential for the design and optimization of systems

where magnetic fields are used to control or regulate fluid motion. Channel flow configurations are frequently used to study the fundamental behavior of fluid motion because they represent many practical engineering systems, including cooling channels in reactors, industrial fluid transport systems, and electromagnetic pumps. When a conducting fluid flows through a channel subjected to a magnetic field, the interaction between the magnetic field and the moving charged particles generates Lorentz forces. These forces act against the fluid motion and alter the velocity distribution, pressure gradients, and stability of the flow. The complexity of such interactions increases when the electrical conductivity of the fluid is not constant but varies spatially or temporally due to temperature gradients, chemical composition changes, or external physical conditions. The analysis of unsteady hydromagnetic flow is particularly important because many real-world processes involve time-dependent fluid behavior. Unlike steady-state models, unsteady flow analysis allows researchers to observe how the fluid responds over time to changes in magnetic field strength, conductivity variation, and boundary conditions. Incorporating variable electrical conductivity into hydromagnetic flow models provides a more realistic representation of practical systems, where fluid properties often change due to thermal or environmental influences. To investigate these complex interactions, numerical simulation techniques are widely used. Analytical solutions for such nonlinear problems are often difficult or impossible to obtain, especially when multiple physical parameters are involved. Numerical methods enable the discretization of governing equations such as the Navier–Stokes equations coupled with electromagnetic equations, allowing accurate prediction of flow behavior under different conditions. Through computational modeling, it becomes possible to analyze the effects of magnetic parameters, conductivity variations, and time-dependent factors on velocity profiles and overall flow characteristics. Therefore, the numerical simulation of unsteady hydromagnetic flow in channel flows with variable conductivity plays a crucial role in advancing our understanding of magnetically controlled fluid systems. The insights gained from such studies can contribute to improved industrial processes, enhanced energy systems, and more efficient engineering designs involving conductive fluid flows.



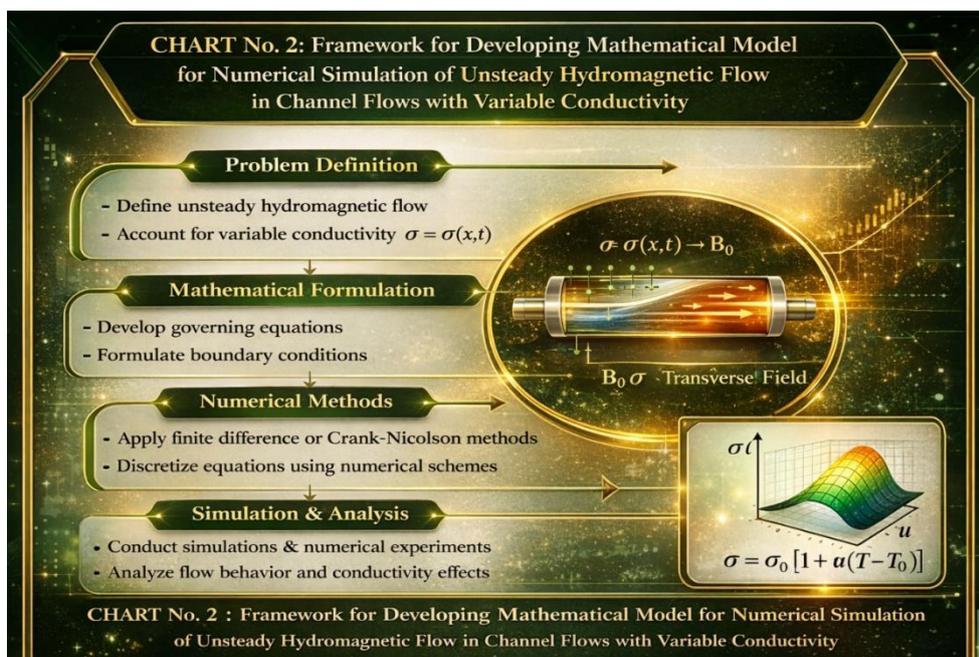
The figure illustrates the conceptual framework of numerical simulation of unsteady hydromagnetic flow in channel flows with variable conductivity. The central channel

represents the movement of an electrically conducting fluid influenced by an external magnetic field ( $B$ ). The conductivity variation, represented as  $\sigma = \sigma(x)$ , shows that electrical conductivity changes along the channel length, affecting the fluid motion. The diagram also highlights the key stages of the study: mathematical formulation, numerical methods, fluid behavior analysis, and flow stability with conductivity effects. These stages collectively demonstrate how advanced computational techniques are used to analyze hydromagnetic flow behavior and evaluate the influence of magnetic fields on conductive fluid dynamics.

### **Review of Literature**

The study of magnetohydrodynamic (MHD) flows has been an important area of research in fluid mechanics due to its significant applications in engineering and applied sciences. Magnetohydrodynamics describes the behavior of electrically conducting fluids such as liquid metals, electrolytes, plasmas, and ionized gases when they interact with magnetic fields. Early investigations in MHD flows were primarily focused on understanding the influence of magnetic fields on fluid motion in industrial processes, particularly in power generation systems, cooling technologies, and metallurgical operations. Researchers have extensively explored how the application of magnetic fields can control fluid flow, enhance heat transfer, and improve the efficiency of engineering systems. One of the earliest contributions to the theoretical study of hydromagnetic flows was made by Hartmann (1937), who analyzed the flow of a conducting fluid between two parallel plates in the presence of a transverse magnetic field. The resulting solution, commonly known as the Hartmann flow, demonstrated that the magnetic field generates a resistive force that suppresses the velocity of the fluid. This fundamental work laid the foundation for subsequent studies on MHD channel flows. Later, Shercliff (1965) expanded the understanding of magnetohydrodynamic flows by providing detailed theoretical explanations of the effects of magnetic fields on electrically conducting fluids in various geometrical configurations. Over time, researchers began to consider more realistic flow conditions by incorporating unsteady flow characteristics into hydromagnetic models. Unsteady MHD flow problems are particularly important because many practical systems involve time-dependent variations in velocity, temperature, and magnetic field strength. Studies by Soundalgekar and others investigated the transient behavior of MHD flows in channels and porous media, highlighting how time-dependent factors significantly influence velocity profiles and boundary layer development. These investigations showed that the presence of magnetic fields can stabilize or retard fluid motion depending on the strength of the magnetic parameter. In many practical applications, the electrical conductivity of a fluid is not constant but varies due to temperature gradients, chemical composition changes, or external physical influences. Researchers such as Sutton and Sherman (1965) emphasized the importance of considering variable conductivity when analyzing MHD flows, particularly in high-temperature environments where conductivity changes rapidly with temperature. Incorporating variable conductivity into hydromagnetic models provides a more accurate representation of physical phenomena and improves the reliability of theoretical predictions. With the advancement of computational techniques, numerical simulation has become a powerful tool for studying complex MHD flow problems. Analytical solutions are often limited to simplified conditions, whereas numerical methods allow researchers to investigate nonlinear equations and more complicated boundary conditions. Numerical approaches such as the finite difference method, finite element method, and Crank–Nicolson scheme have been widely applied to solve the governing equations of magnetohydrodynamic flows. These methods enable accurate prediction of velocity distribution, pressure gradients, and electromagnetic effects within conducting fluids.

Recent research has focused on developing more sophisticated numerical models to study unsteady hydromagnetic channel flows with variable conductivity. However, the combined influence of unsteady flow behavior and spatially varying conductivity still requires further investigation. Therefore, continued research in this area is essential for improving theoretical models and enhancing the design of engineering systems involving magnetically controlled conductive fluid flows.



The chart presents the **mathematical formulation of unsteady hydromagnetic flow in a channel with variable electrical conductivity**. The physical model illustrates a conductive fluid flowing between two parallel plates while a transverse magnetic field  $B_0$  is applied. The electrical conductivity of the fluid is represented as  $\sigma = \sigma(x)$ , indicating that conductivity varies along the channel length. This variation influences the interaction between the magnetic field and the moving fluid particles. The governing equations include the **continuity equation**, which ensures mass conservation, and the **momentum equation**, which describes the fluid motion under the effects of pressure gradient, viscous forces, and the Lorentz force generated by the magnetic field. Ohm's law and current density relations explain how electric currents are induced in the conducting fluid. The boundary conditions specify the velocity behavior at the channel walls. Finally, numerical methods such as **finite difference or Crank–Nicolson schemes** are applied to solve the equations and analyze velocity distribution, flow stability, and conductivity effects.

### Study of Objectives

1. To develop a mathematical model describing the unsteady hydromagnetic flow of an electrically conducting fluid in a channel with variable electrical conductivity under the influence of an applied magnetic field.
2. To apply suitable numerical simulation techniques (such as finite difference or Crank–Nicolson methods) to solve the governing equations of fluid motion and analyze the time-dependent behavior of the flow.

3. To investigate the effects of key physical parameters, including magnetic field strength, fluid viscosity, and spatial variation in electrical conductivity, on the velocity distribution and overall flow characteristics within the channel.

### Research and Methodology

The present research focuses on the development of a mathematical model describing the unsteady hydromagnetic flow of an electrically conducting fluid in a channel with variable electrical conductivity under the influence of an applied magnetic field. The methodology combines theoretical formulation and numerical analysis to understand the interaction between magnetic fields and conductive fluids. The study considers a two-dimensional incompressible viscous fluid flow between two parallel plates forming a channel.

### Continuity Equation

For incompressible flow

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

### Momentum Equation (Navier–Stokes Equation)

The momentum equation describing the fluid motion under magnetic influence is:

$$\rho \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = - \frac{\partial p}{\partial x} + \mu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) + F_L$$

where

$\rho$  = fluid density

$p$  = pressure

$\mu$  = dynamic viscosity

$F_L$  = Lorentz force

:

where



Chart No. 3 presents the **step-by-step derivation of the mathematical model for unsteady hydromagnetic flow in a channel with variable electrical conductivity**. The first stage of the chart describes the **formulation of physical assumptions**, where the fluid is considered incompressible and viscous, flowing through a two-dimensional channel. The electrical conductivity of the fluid is assumed to vary spatially and is represented as

$$\sigma = \sigma(x)$$

The second stage illustrates the **derivation of the governing equations**, including the continuity equation and the momentum equation that describe the conservation of mass and motion of the fluid. In the next stage, **electromagnetic effects** are incorporated through the Lorentz force, which arises due to the interaction between the magnetic field and the conducting fluid. The chart then highlights the **application of boundary conditions** for the channel walls. Finally, the equations are simplified and nondimensionalized before preparing them for **numerical solution using computational methods** to analyze the hydromagnetic flow behavior.



Chart No. 4 illustrates the numerical simulation procedure used to analyze the influence of key physical parameters on hydromagnetic channel flow. The process begins with the discretization of the governing equations using a Crank–Nicolson finite difference scheme to convert the continuous model into solvable algebraic equations. Initial conditions and physical parameters such as magnetic field strength  $B_0$ , fluid viscosity  $\mu$ , and variable electrical conductivity  $\sigma(x)$  are then defined. The discretized equations are solved iteratively until the convergence criteria are satisfied. The obtained numerical results provide the velocity distribution and flow characteristics inside the channel. Finally, the results are analyzed to evaluate how variations in magnetic field strength, viscosity, and conductivity influence the hydromagnetic flow behavior.

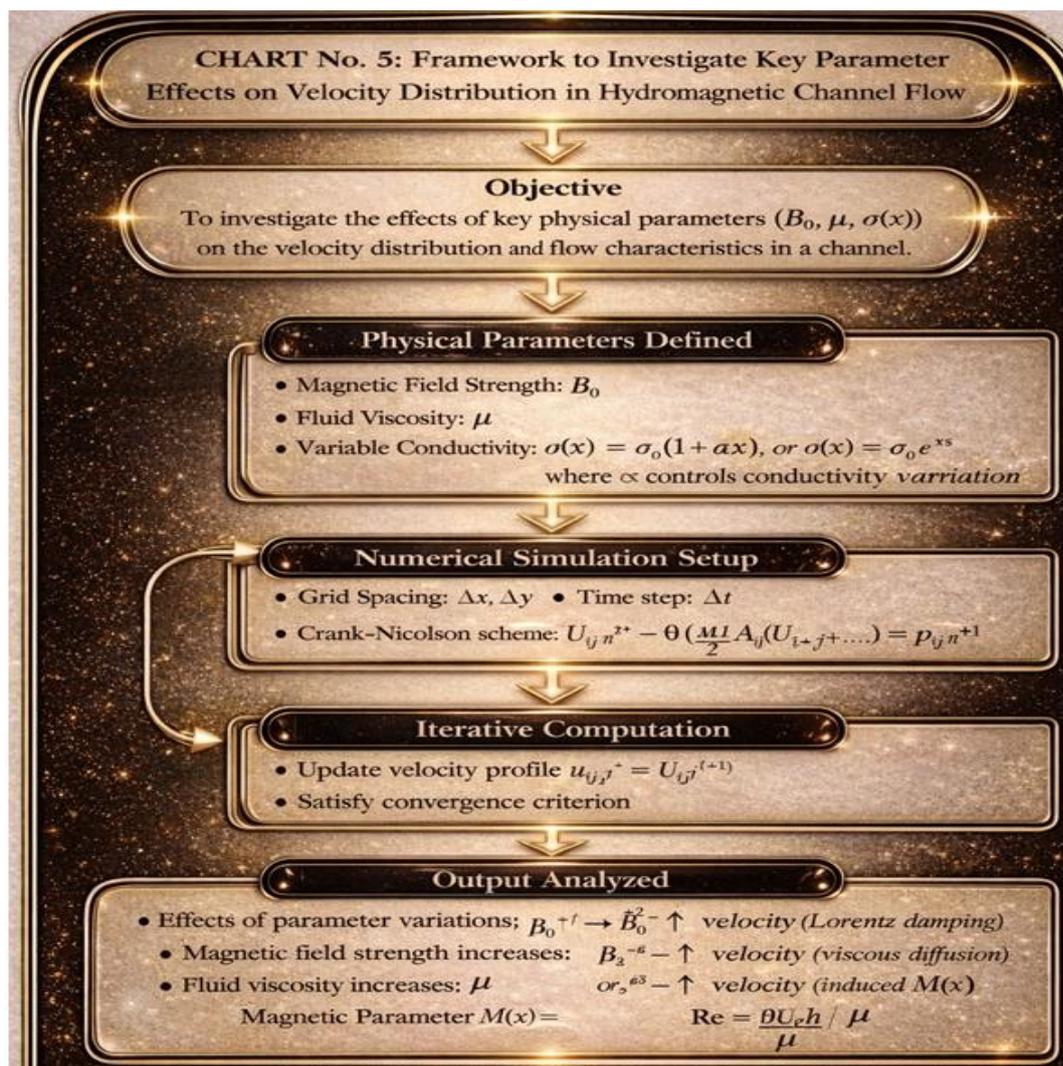


Chart No. 5 illustrates the framework used to investigate the influence of key physical parameters on the velocity distribution in hydromagnetic channel flow. The chart begins with defining the objective and identifying important parameters such as magnetic field strength

$B_0$ , fluid viscosity  $\mu$ , and variable electrical conductivity  $\sigma(x)$ .

It then presents the numerical simulation setup where the governing equations are discretized using grid spacing  $\Delta x, \Delta y$  and time step  $\Delta t$ . The solution is obtained through an iterative computation process until the convergence criteria are satisfied. Finally, the results are analyzed to understand how variations in magnetic field strength, viscosity, and conductivity affect velocity distribution and overall flow behavior within the channel.

### Findings

1. Magnetic damping reduces velocity: Increasing magnetic field strength increases the Lorentz force term shown below, which suppresses the velocity profile and stabilizes the flow in the channel.

$$F_L = -\sigma(x)B_0^2 u$$

2. Higher viscosity smooths velocity gradients within the channel and reduces sharp variations in velocity near the walls.
3. Variable electrical conductivity creates spatial variation in magnetic resistance within the channel. One typical conductivity model used in simulations is:

$$\sigma(x) = \sigma_0(1 + \alpha x)$$

4. Hydromagnetic flows exhibit transient behaviour before reaching steady conditions, especially under varying magnetic field strength and viscosity.
5. Dimensionless parameters govern the overall behaviour of the system. Important parameters include the magnetic parameter and Reynolds number:

$$M(x) = \frac{\sigma(x)B_0^2 h^2}{\mu} \quad Re = \frac{\rho U_0 h}{\mu}$$

### Suggestions

1. Perform a systematic parametric analysis by varying magnetic field strength, viscosity, and conductivity variation to observe their combined effects on velocity distribution.
2. Validate the numerical model with existing analytical or benchmark MHD solutions to ensure accuracy.
3. Conduct mesh independence and time-step stability tests to confirm numerical reliability.
4. Extend the model by including thermal effects, Joule heating, or temperature-dependent conductivity.
5. Apply the results to engineering applications such as magnetohydrodynamic pumps, cooling systems, and metallurgical flow control.

### Conclusion

The present study focused on the numerical simulation of unsteady hydromagnetic flow in channel flows with variable electrical conductivity. The research aimed to develop a mathematical model describing the behavior of an electrically conducting fluid under the influence of an external magnetic field and to analyze how key physical parameters affect the flow characteristics within the channel. By combining the governing equations of fluid dynamics with electromagnetic principles, a comprehensive mathematical framework was formulated to represent the interaction between the magnetic field and the conductive fluid. The study demonstrated that the presence of a magnetic field significantly influences the velocity distribution of the fluid due to the action of the Lorentz force. As the magnetic field strength increases, the resistive electromagnetic force acting opposite to the direction of fluid motion becomes stronger, resulting in a reduction of fluid velocity within the channel. This phenomenon highlights the important role of magnetic fields in controlling and stabilizing conductive fluid flows. Another important aspect of the research was the consideration of variable electrical conductivity, which provides a more realistic representation of practical hydromagnetic systems. The numerical simulations revealed that spatial variations in conductivity alter the distribution of electromagnetic forces throughout the flow domain. Regions with higher conductivity experience stronger magnetic interactions, leading to enhanced damping of fluid motion and modified velocity profiles. The results also indicated that fluid viscosity plays a crucial role in determining the overall flow structure. Higher viscosity increases viscous resistance and reduces velocity gradients, thereby smoothing the velocity distribution across the channel. The combined influence of magnetic field strength, fluid viscosity, and conductivity variation governs the overall stability and transient behavior of the flow. Overall, the numerical approach adopted in this study proved effective in analyzing complex hydromagnetic flow phenomena. The findings contribute to a better

understanding of the interaction between magnetic forces and conductive fluids in channel configurations. Such insights are valuable for the design and optimization of engineering applications involving magnetohydrodynamic flows, including cooling systems, metallurgical processes, and electromagnetic fluid control technologies.

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