European Economic Letters
ISSN 2323-5233
Vol 13, Issue 5 (2023)
http://eelet.org.uk

A Study of The Effects of Classroom Structure and Instructional Methods on Students Learning of Trigonometric Concepts in Mathematics

Dr Puneet Kaur[1]*

[1]*Assistant Professor, D.D. Jain College of Education, Ludhiana (Punjab) India-141008

***Corresponding Author:** Dr Puneet Kaur

*Assistant Professor, D.D. Jain College of Education, Ludhiana (Punjab) India-141008,
E-mail Id: puneet916@gmail.com

Abstract:
To think in a methodical and logical way, mathematics is an essential skill. On top of that, it may be used to learn new things in many other subjects. Recently, there has been a need to find better ways to teach mathematics, especially trigonometry, at senior high schools (SHSs), in order to increase students' understanding of the subject. Thus, numerous studies on what makes a good teaching style that helps students understand the material are necessary. Research has shown that students in higher education, particularly those at the secondary school level, suffer when teachers are unable to relate to the elements of a meaningful-based teaching approach. This study sheds insight on the ways in which a student's learning environment and instructional approaches affect his or her capacity to learn trigonometry, an area where academic policy-makers and educators are now developing transformational initiatives.The study objectives of this article are founded on the premise that environmental motivating characteristics are necessary to excite students' attention and intellect when learning trigonometry, a real-life method that depends on the size and distance of an object. As a result, this program aims to improve and make better use of two features—the classroom environment and instructional method—in elementary and secondary school settings.

Keywords: trigonometry learning, brain based teaching, active teaching, visual trigonometry, symbolic trigonometry

1. Introduction

Mathematics is a crucial discipline that enables humans to engage in systematic and logical reasoning. Additionally, it serves as a tool for acquiring knowledge in various fields of study. Hence, mathematics plays a crucial role in the advancement of humanity. To facilitate the development of skills outlined in the curriculum, it is necessary to depend on many elements. The primary factor of significance is the educational endeavors facilitated by instructors.

Teachers must possess a comprehensive understanding of the curriculum's objectives, as well as a deep comprehension of various problem types. Additionally, they should possess the ability to analyze and effectively resolve any issues that may arise during classroom sessions. The Ministry of Education proposed a set of recommendations for organizing educational activities in the implementation of this curriculum: The kid is positioned at the core of the learning process, thinking process, problem-solving process, value development process, skill-building process, and with a focus on group dynamics (Vihokpaibul, 2018).

In recent years, the pursuit of effective teaching techniques is necessary to stimulate students' comprehension of mathematics education, namely in the field of trigonometry in senior high schools (SHSs). Therefore, prompting the need for several research studies on the characteristics of an effective teaching method that enables pupils to comprehend the subject matter. Prior study suggests that instructors' lack of capacity to connect with the components of a meaningful-based teaching method has an impact on students in postsecondary institutions, especially at the SHS level. There is a lack of consensus over a singular method for teaching mathematics (Asomah et al., 2023).

On this issue, where educators and academic policy-makers are currently building up transformational plans, this research casts light on the impacts on a student's trigonometric learning ability with his/her learning environment and instructional methods. It is relied while formulating the study objectives of this article that trigonometry, being a real life based approach dependent on size and distance of an object needs environmental motivational parameters to stimulate student's attention and intellect. Thus, classroom environment and teaching style are the two training features that are specifically focused and explored looking for their betterment and suitable utilization in primary and secondary school levels.

2. Background of the Study

In real-life scenarios, as is commonly observed, the learning activities in mathematics classrooms have not adhered consistently to the objectives and rules of the curriculum. This was due to the predominant use of lecture-based instruction by most professors. The kids were neither motivated or prompted to engage in critical thinking. Teachers failed to recognize the significance of lesson planning.

Some instructors tend to teach at a rapid pace, without acknowledging the unique characteristics and needs of each individual. Teachers lacked sufficient time to adequately prepare their classes. Their instruction prioritized the end results rather than the procedural steps. The inclusion of such learning activities in mathematics classrooms has a significant impact on the learners in several ways.

For instance, learners exhibit a deficiency in comprehending procedures and struggle to maintain a consistent knowledge of the courses. This results in subpar mathematics instruction and learning at the elementary level, falling short of the effectiveness outlined in the curriculum. According to real learning and teaching activities, academic performance in mathematics is now at an unsatisfactory level.

During mathematics lessons, students were not entrusted with the task of finding solutions independently, which hindered the optimization of their cognitive abilities. Typically, the more intelligent students were consistently given the opportunity to showcase their abilities at the front of the class, while those who were less academically inclined were discouraged from engaging in critical thinking and were instead assigned the job of resolving simpler problems.

The low motivation of students in learning mathematics is evident from their lack of seriousness in participating in class and the majority of students expressing dissatisfaction with the teaching methods, whether it be individual or group instruction. This is further supported by the low scores on students' daily tests.

An alternative teaching process, named, Brain-based learning refers to a learning approach that is in harmony with the functioning of the brain and is naturally built to facilitate learning. Brain-based learning is an educational strategy that is grounded in ideas gained from a comprehensive understanding of the brain.

There are some brain structures that are primarily involved in learning based on principles of design. The brain provides a framework for developing a learning approach that focuses on empowering students' cognitive abilities. Jensen outlines the process of brain-based learning as follows: (1) pre-exposure, (2) preparation, (3) initiation and acquisition, (4) elaboration, (5) incubation and encoding of memory, (6) verification and trust checking, and (7) celebration and integration. Rose suggests that one must possess a mindset that is abundant in perception. It implies that you need to be calm, self-assured, and driven. If you experience stress or lack confidence, you may not perform effectively (Mekarina & Ningsih, 2017).

Here, we present a current days' evidence to strengthen our debate on the justification of brain-based learning to be more effective in learning trigonometry. According to Quaisley& Wakefield(2022), active learning is a crucial element in several collegiate Science, Technology Engineering, and Mathematics (STEM) courses, notably mathematics. Active learning-oriented mathematics classrooms are designed to foster students' participation in computational thinking and interaction between.

In active learning classrooms, teachers prioritise fostering student involvement and leveraging student cognition as students generate inquiries, articulate logical reasoning, and exchange answers. In contrast, we see lecture-oriented classrooms as being distinguished by conventional, teacher-focused teaching methods. Freeman and colleagues (2014) conducted a meta-analysis of 225 papers and discovered that student performance on tests was notably superior in active learning classrooms as opposed to programmes that prioritise traditional lecturing.

According to research, active learning has been found to potentially decrease achievement disparities among minority students in STEM courses when compared to standard lecturing. Hence, active learning possesses the capacity to impartially enhance student achievement in undergraduate mathematics courses.

Again, an essential determinant of the teaching and learning process is the development of efficient instruction that can cater to a wide range of learning styles and academic backgrounds. Evidence of learning may be observed not just via changes in behavior, but also through changes in cognitive processes. Efficient student learning is a direct outcome of proficient teaching techniques, together with the teacher's comprehensive understanding of the subject matter.

In addition, the clarity of a teacher, their ability to stimulate interests, and their openness to perspectives are additional important aspects that contribute to the management of learning efficiency and effectiveness. The teacher's critical comprehension of learning theories and their application to the cognitive, motivational, and psychological aspects of the learning process is crucial for academic achievement.

Within open and flexible learning environments, instructional materials provide the capability to accommodate individual requirements while facilitating collaborative modes of learning. Initially, while creating educational materials for a specific group of learners, instructional designers usually do requirements analyses or learner profile to determine the learners' existing knowledge, motivations, background interests, attitudes, and experiences. The reason for conducting such research is that individual predispositions can significantly impact learners' receptiveness to teaching and their academic advancement.

Instructional designers often recognize and accommodate individual variations in their designs, and frequently strategize to tailor training to the specific requirements of each student. The process of analyzing the requirements of learners allows for the development of learning materials that are specifically designed to meet those needs (Misheck, 2016).

3. Related Works

In this section, a systematic review on the latest academic articles is provided including trigonometry teaching procedures, their difficulties and rectifying models. The section particularly emphasizes on the approaches of active or brain-based teaching process making use of interactive and open-flexible teaching procedures rather than lecture based pedagogical procedures as are followed traditionally. This is done because these traditional processes are found to be suffering from bias and lack of motivation on the students and thus need transformations.

Husna et al. (2018)assessed the disparities in students' mathematical connection skills before and after the adoption of the Brain-Based Learning (BBL) model. Here, the author examined the variations in the enhancement of students' mathematical connection abilities between the implementation of BBL models and traditional direct learning. The process evaluation was done to rate the suitability of BBL models and student engagement in trigonometry activities in class X Senior High School (SMA) in Singkawang.

The findings indicated the presence of noteworthy disparities in students' mathematical connection aptitude when employing BBL models. Furthermore, disparities were observed in the enhancement of mathematical connection abilities between students who received BBL model instruction and those who received direct model learning. The implementation of the BBL model in class X Senior High School was highly successful, and student learning activities were deemed active when the BBL model was applied in Senior High School.

Nanmumpuni&Retnawati (2021)examined the challenges associated with solving trigonometry conceptual issues among senior high school students across various levels of creative thinking ability. This study employed a descriptive methodology with the objective of identifying the challenges and their underlying causes encountered by a group of five students. The challenges are linked to the idea of trigonometry and its accompanying complications. The findings indicated that students have challenges in comprehending the notion of trigonometry across various levels of cognitive abilities. The challenge is in recognizing the practical use of trigonometric principles in everyday situations and understanding the connections between the supplied challenges.

Spangenberg (2021), in his exploratory case study referred to the inferences of Hill et al.'s (2008) mathematics knowledge for teaching (MKT) approach. According to the author, the model served as the foundation for active teaching procedure. The results found in this research demonstrated how teachers applied the four components of pedagogical content knowledge (PCK) on trigonometry—knowledge of the subject, knowledge of teaching techniques, knowledge of students' concepts, and knowledge of the curriculum—though to varying degrees of adequacy. He argued that understanding these variations provided a solid foundation for organizing professional development activities that could improve instructors' PCK on trigonometry.

As proposed by **Mesa &Herbst (2021)**, it is as open and flexible learning environments, instructional materials provide the capability to accommodate individual requirements while facilitating collaborative modes of learning. When creating resources for a specific group of learners, instructional designers usually start by doing a requirements analysis or learner profile. The author stressed that this helps them determine the learners' past knowledge, motivations, background interests, attitudes, and experiences. Furthermore, he explained that the reason for doing these researches is that individual characteristics will somehow impact and frequently shape learners' preparedness to benefit from the education provided, as well as influence their academic advancement.

Harden & Jones (2022)presented the validation of brain-based learning as a conceptual framework to comprehend the influence of brain development and its perceptual stimulations on learning and teaching.The researchers established that the procedure fulfilled the field of social work education. Brain-based learning, as the authors discussed was a sophisticated method that drew on theoretical frameworks from education, neuroscience, psychology, biology, and medicine. Here, the application of the fundamentals of brain-based learning had been linked to enhanced motivation, enhanced academic achievement, and enhanced knowledge retention. The experiential practises ingrained in social work pedagogy are supported by the brain-based learning model, which is highly suited for use in social work education.

Canonigo (2023)discussed with the help three different classroom scenarios thathow educators work together to interact and develop a curriculum by means of problem-solving techniques. The author argued that collaborative environment provided ample scope to help students grasp the concepts of trigonometry, such as, concept of sine, cosine, and tangent. Through problem-solving-based arithmetic instruction, students were able to use their existing knowledge, consider theirremedies to the issue, and interpret their responses. In this work, the researcher showed that the pupils were able to communicatetheir prior knowledge and abilities as they collaborated to comprehend the mathideas present in the issue. As a result, they were able to offer various fixes for the issue.

Still, it is advised that different teaching techniques should be used and the teacher was still in charge of deciding which strategies were best for a given idea. The author stressed that it was crucial for an instructor to understand how to introduce mathematical concepts using the responses and solutions provided by the pupils to the problem. Therefore, in this work, it is specifically indicated that math teachers needed to be able to draw links between different topics and comprehend the underlying meanings and justifications of concepts, especially when they come from the students.

The literatures that are reviewed above provide a clear indication that students are much more psychologically attentive towards participatory learning rather than instructive and theory based explanations. Currently, a number of methods are being tried to increase student participation and motivating their stimulation towards knowing the concepts. Among them Brain-Based Learning and Collaborative Learning have received much attention and are being applied in school and higher academic institutes. Particularly, teaching trigonometry needs deeper association with physical specimens in order to grasp its concepts and thus needs subject specific studying environment and teaching methods. The current researchers as they are reviewed here are at their evolving stage in this matter and need more extensive and assessment based analysis to come up with the most suiting technique on teaching trigonometry.

4. Conceptualization and Methodology

Based on the resources as obtained from the current existing academic works on brain based and collaborative learning that are both considered relevant to construct the base concept of this research, here these two techniques are integrated logically to deliver the lesson. Side-by-side, attempts are made to test and rate student's motivation level. As already identified these techniques need assessment in terms of student's perception development and thereby to be implemented strategically, considering two aspects, namely: (a) Student's psychological motivation towards the subject; and (b) Teacher's capacity to gather student's attention and willingness to learn, irrespective their intelligence level and interests. Base principle followed in this approach centralizes on human brain's learning processes and motivational channels. According to the definition presented by Caine & Caine (1991, p. 4), brain-based learning is "organizing teaching with those rules in mind and acknowledging the brain's rules of meaningful learning." There is disagreement, meanwhile, regarding the nature of the brain principles that underpin meaningful learning and the best ways to structure instruction to make use of them. The following list of guidelines for brain/mind learning was put up by educational scholar Caine and learning consultant Caine (1997) (Tait, 2007).

Principle 1: The brain is an intricate system of adaptation.
Principle 2: The social brain is the brain.
Principle 3: People are born with a yearning for purpose.
Principle 4: Patterns help in the search for meaning.
Principle 5: Patterning depends on emotions.
Principle 6: Parts and wholes are concurrently perceived and created by every brain.
Principle 7: Both focused attention and peripheral perception are necessary for learning.
Principle 8: Both conscious and unconscious processes are constantly involved in learning.
Principle 9: Our memory is organised in at least two different ways.
Principle 10: Development occurs throughout learning.
Principle 11: Challenge and inhibited threat improve complex learning.
Principle 12: Every brain is structured differently. Page 19

Caine & Caine (1997) attempted to integrate these ideas with conventional teaching methods. Three different forms of teaching technique were described in the Caine and Caine (1997) study. There are five components to any model: (a) The goals of education;(b) How teachers use their time;(c) Where they find curriculum and instruction; (d) how they define and handle discipline; and (e) how they handle evaluation are the five areas of focus for teachers (p. 216).

5. Findings and Discussion

Coming to the actual learning environment, a conceptual knowledge, or grasp of the trigonometry notion, is one of the key competences after learning trigonometry. Students that possess a conceptual knowledge are able to integrate concepts and procedures and use them to solve difficulties in their everyday lives. The instructional design idea known as Cognitive Load idea (CLT) is predicated on our understanding of the cognitive organization of humans.

According to CLT theory, in order to promote effective learning, instructional designs should take cognitive structure into account. The foundation of CLT is the idea that when new learners encounter difficult content, their working memory will become less capable of holding information for extended periods of time, as they will lack the foundational knowledge to arrange it. On the other hand, knowledge stored in long-term memory can be recovered and stored in working memory indefinitely.

Learning strategies that help students manage their cognitive resources so that learning does not tax their working memory and schema are built and organized in long-term memory are the main focus of the CLT. Let us look into the different CLT techniques used in trigonometry teaching process.

- **Verbal interaction:**One of the many instructional designs based on CLT that has been developed is goal-free problems. Goal-free problems reduce unnecessary cognitive strain and enhance transferability in problem-solving, according to Ayres' research.

  Goal-free issues are characterized by the substitution of an open question, such as "evaluate the length of all unknown segments," for a specific inquiry, such as "evaluate the length of the segment a" in geometry. Instead of employing a means-ends analysis or a going backward technique, it has been discovered that when students are given goal-free problems to solve, they solve the problem by comprehending the provided knowledge to the unknown variables.

  Again, apart from verbal interactions, mathematicians have specified a variety of ways that quantitative concepts might be communicated to individual students since different people learn in various ways(Macdonald, 2009).
- **Graphical/Visual Representation:** Visual representation is particularly helpful for those students who are aware of geometric basics and needto be trained so that they can easily relate and understand the connectivity between geometry and trigonometry. Example:

  Trigonometric Functions should be visualized by using a standard unit circle (a circle with radius 1 unit). Here, we have $\angle XOP = \theta$, the angle in standard position and $P(x, y)$ be at the terminal position such that, $l(OP) = r > 0$,

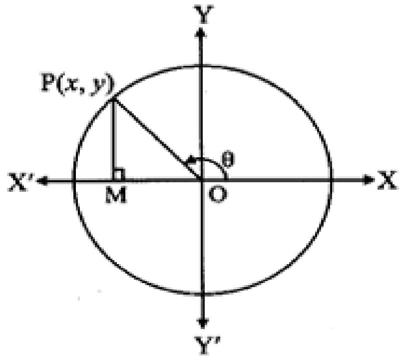

Figure 1: Visual Representation of Trigonometric Functions in unit circle
(Source: DGT (2017))

Then, we have, $sin\theta = \frac{y}{r}$, $cos\theta = \frac{x}{r}$, $tan\theta = \frac{y}{x}$ $(x \neq 0)$
This learning can be further enhanced through animated visual aids and AI based virtual reality. This method is used for improving the perception, retention, and replication of associations and pictures generated by transformation and binding processes.
Another example is given below that shows the sign convention of trigonometric functions by using a unit circle:
According to the CAST Rule, all three $sin\theta, cos\theta,$ and $tan\theta$ values in quadrant I are positive. Only $sin\theta$ is positive in quadrant II; $cos\theta$ and $tan\theta$ are negative. Only $tan\theta$ is positive in quadrant III; $sin\theta$ and $cos\theta$ are negative. Only $cos\theta$ is positive in quadrant IV; $sin\theta$ and $tan\theta$ are negative. Just start at the bottom right quadrant and mark the quadrants C-A-S-T, working your way clockwise, to help you remember this(SFU, 2021).
This signing convention, when represented in unit circle, lets the student visualize the rotation and learn the CAST sign rule.

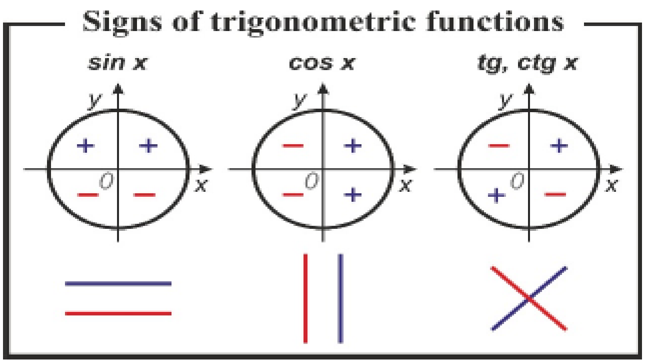

Figure 2: Visual Representation of trigonometric signing convention in unit circle
(Source: Drushlyak et al. (2021))

- **Symbolic Representation:** This method is a traditional procedure of explaining trigonometry and now is mostly aided with visual representation to enhance its clarity. Example: The fundamental equation of trigonometric identity, like:

$Sin^2\theta + Cos^2\theta = 1$; $1 + Tan^2\theta = Sec^2\theta$ …… and more.

Again, when we represent it with unit circle, it lets students to visualize the actual condition and hence becomes easy for them to understand the validity of these identities.

As you can see that, here the terminal point of the radius of the unit circle that lies on the circle boundary is represented as (cosθ,sinθ). It is understandable from the previous explanation of formulae that when r = 1 unit, Cosθ = x, Sinθ = y
Hence, by Pythagorus Theorem,
$x^2 + y^2 = 1$
or, $Cos^2\theta + Sin^2\theta = 1$
Now, dividing $cos^2\theta$ to both sides of the equation, we get,
$1 + Tan^2\theta = Sec^2\theta$

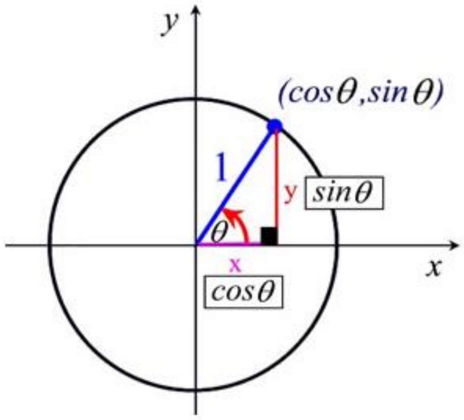

Figure 3: Visual Representation of trigonometric identities in unit circle (Source: Roberts & Roberts (2022))

• **Physical Representation:** In this method, the teacher explains a problem by using physical props and samples.

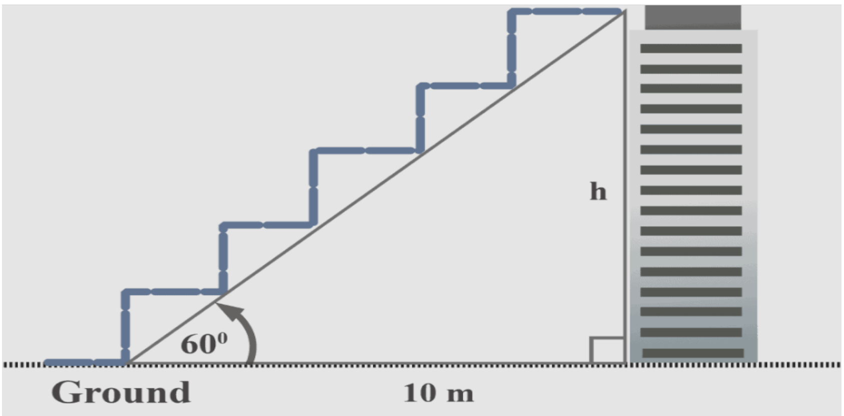

Figure 4: Physical Demonstrative Representation of a trigonometricproblem
(Source: Smith(2023))

In the above illustration, a trigonometric problem is demonstrated by using physical model. We can see that the problem asks to find the height of a building, where a staircase is built at a distance of 10 m away from it and it makes 60^{o} angle with the ground. You can see that these problems are also visual representation but they are much more relating to real life components than the geometry-based visual representations. Such representations are used to teach the students who are completely new to geometry and their geometric perception is not so strong.

Now, the teaching techniques that we've discussed above clearly suggest that meaningful knowledge of mathematical concepts and relationships are enhanced by the use of numerous representations, which convey several forms of the same mathematical situation and highlight the relationship between the complex concepts. Many in the field of mathematics education believe that students learn mathematics best when presented with a variety of representations (NCTM, 2000). When it comes to mathematical reasoning and problem comprehension, representations are crucial. Students' ability to think dynamically and creatively about problems is enhanced through the use of many representations, which in turn aids their comprehension of mathematical topics.

We can see that trigonometric problems are best taught by using three forms of training instructions: Visual, Symbolic and Verbal(Dündar, 2015). There are past researches that have evaluated these teaching concepts. Example: Research by Akyüz, Coşkun, and Coşkun (2009) on the impact of multiple representations on the problem-solving abilities of teacher-candidates found that, when faced with a problem, the candidates relied on mental images; in the absence of such an image, they struggled to find a solution. When Hammill (2010) looked at math textbooks from a verbal, symbolic, and visual perspective, she found that they had all three. However, she stressed the need of checking to see if the appropriate connections were made between these representations.

Kar and İpek (2009) demonstrated the necessity of using visual representations in mathematics education and problem solving by researching their usage in solving verbal problems across mathematical history and discovering that they were an effective strategy for comprehending and resolving problems. According to Villegas et al. (2009), students need to be able to connect different kinds of representations in order to solve problems. The study identified a strong correlation between students' problem-solving performance and their capacity to switch between visual and symbolic representations. In practice, the work of Ngu&Phan (2023)suggests that visual designs of a problem motivate the students more than unguided problem-solving approach that includes verbal interaction. Physical representation based problem solving is also an effective method for new students who lack in mathematical conceptualization. This method provides them analogy that they use to relate their problem with real life condition.

6. Conclusion

This article discussed by means of multiple existing researches and observations that it is difficult for teachers to train the students Trigonometry, particularly those who are new to the subject and do not have sufficient clarity on basic geometric concepts. Since trigonometry is connected to real life components, such as, distance, height and size, visual representation is found the most suiting technique to train the students. Accordingly, there are modern tools and devices available that should be tried to teach the learners. Example: Physical dummy models, AI based tools and apps and geometric models are recommended accessories that the teachers can make use of. Lesson planning is very important for trigonometric training. In this article, brain-based and collaborative approach is emphasized. The reason behind choosing these two approaches is that rather than theoretical explanations, students show more willingness and interest to learn when they are included in the training session and made to participate in the teaching process, irrespective of their intelligence and knowledge. Such a procedure is found to enrich a student's perception and motivation in problem-solving. However, verbal problem solving mode should be supported by visual representations as these representations are found to stimulate brain's perception functions more than oral interactions.

References

1. Asomah, R. K., Agyei, D. D., Ntow, F. D., &Benning, I. (2023). Hypothetical approach to the teaching of Trigonometric Concepts Using Cooperative Learning. *Education Research International*, *2023*, 1–12. https://doi.org/10.1155/2023/2051776
2. Canonigo, A. M. (2023). Three Teachers, One Lesson on Teaching Trigonometry through Problem Solving in a Lesson Study. *Learning More Together, Growing in Practice Togather*, (2), 1–25.
3. DGT. (2017). *03 trigonometric functions - dgtstudy*. dgtstudy.com. https://dgtstudy.com/media/media/study_material/2020-04-06/DGT_Trigonometric_Functions.pdf
4. Drushlyak, M. G., Semenikhina, O. V., Proshkin, V. V., &Sapozhnykov, S. V. (2021). Training pre-service mathematics teacher to use mnemonic techniques. *Journal of Physics: Conference Series*, *1840*(1), 012006. https://doi.org/10.1088/1742-6596/1840/1/012006
5. Dündar, S. (2015). Mathematics teacher-candidates' performance in solving problems with different representation styles: The trigonometry example. *EURASIA Journal of Mathematics, Science and Technology Education*, *11*(6). https://doi.org/10.12973/eurasia.2015.1396a
6. Harden, V., & Jones, V. N. (2022). Applying the principles of brain-based learning in Social Work Education. *Advances in Social Work*, *22*(1), 145–162. https://doi.org/10.18060/25142
7. Husna, N., Nurhayati, P., N. C., Wahyuni, R., Utami, C., Rosmaiyadi, Mariyam, &Kartina. (2018). Implementation of brain-based learning model to increase students' mathematical connection ability on trigonometry at senior high school. *Proceedings of the 2018 2nd International Conference on Education and E-Learning*. https://doi.org/10.1145/3291078.3291103
8. Macdonald, H. (2004) *Geologic Puzzles: Morrison Formation*, Starting Point. Retrieved Sept 9, 2004, from http://serc.carleton.edu/introgeo/interactive/examples/morrisonpuzzle.html
9. Mekarina, M., &Ningsih, Y. P. (2017). The effects of brain based learning approach on motivation and students achievement in Mathematics learning. *Journal of Physics: Conference Series*, *895*, 012057. https://doi.org/10.1088/1742-6596/895/1/012057
10. Mesa , V., &Herbst , P. (2021). Designing representations of trigonometry instruction to study the rationality of community college teaching. *Deepblue Library*, 1–33.
11. Nanmumpuni, H. P., &Retnawati, H. (2021). Analysis of senior high school student's difficulty in resolving trigonometry conceptual problems. *Journal of Physics: Conference Series*, *1776*(1), 012012. https://doi.org/10.1088/1742-6596/1776/1/012012
12. Ngu, B. H., &Phan, H. P. (2023). Differential Instructional Effectiveness: Overcoming the challenge of learning to solve trigonometry problems that involved algebraic transformation skills. *European Journal of Psychology of Education*, *38*(4), 1505–1525. https://doi.org/10.1007/s10212-022-00670-5
13. Quaisley, K., & Wakefield, N. (2022). From active learning trigonometry to lecture-oriented calculus: Student interactions. *The Nebraska Educator*. https://doi.org/10.32873/unl.dc.ne034
14. Roberts, D., & Roberts, F. (2022). Pythagorean identities – math bits note book(a2). https://mathbitsnotebook.com/Algebra2/TrigConcepts/TCPythIden.html
15. SFU. (2021). Trigonometry. https://www.sfu.ca/math-course notes/ Math% 20157 %20 Course% 20 Notes/ sec_ Trigonometry.html#:~:text=The%20CAST%20Rule%20says%20that,tan%CE%B8%20%E2%81%A1%20are%20negative.
16. Smith, W. (2023, February 7). *Applications of trigonometry - explanation & examples*. The Story of Mathematics. https://www.storyofmathematics.com/applications-of-trigonometry/
17. Spangenberg, E. D. (2021). Manifesting of pedagogical content knowledge on trigonometry in teachers practice. *Journal of Pedagogical Research*, *5*(3), 135–163. https://doi.org/10.33902/jpr.2021371325

18. Tait, C. (2007). *Trigonometry unit based on Brain Research*. ePublications at Regis University. https://epublications.regis.edu/theses/77
19. Vihokpaibul, P. (2020). Brain-Based Learning Materials and Mathematics Learning Skills of Primary 1 –3 Students. *Palarch's Journal Of Archaeology OfEgypt/Egyptology*, *17*(9).